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MEETING OF 1935 MAY 10

Mr. J. H. Reynolds, President, in the Chair.

The President announced that the Society's Address to His Majesty the King on the occasion of his Silver Jubilee had been sealed and signed by himself and the two Hon. Secretaries at a special meeting of the Council and transmitted to His Majesty. The following reply to the Address had been received from His Majesty through the Secretary of State for the Home Office :—

HOME OFFICE,
WHITEHALL.
1935 *May* 7.

Sir,—I am directed by the Secretary of State to say that he has been commanded by the King to convey to you His Majesty's thanks for the loyal and dutiful Address from the Royal Astronomical Society on the completion of the Twenty-fifth Year of His Majesty's Reign and to assure you that His Majesty deeply appreciates the sentiments of loyalty and affection to which it gives expression.—I am, Sir, Your obedient Servant,

H. A. STRUTT.

THE SECRETARY,
The Royal Astronomical Society,
Burlington House, W. 1.

The President further announced that His Majesty the King had conferred on him, as President of the Society, the Silver Jubilee Medal.

The election by the Council of the following Fellows was duly confirmed :—

Henry Robert Broadhurst, White Cottage, Lime Grove, Eastcote, Ruislip, Middlesex ;
C. R. Burch, Imperial College of Science, S. Kensington, S.W. 7 ;
William MacIntyre, 70 Pinner View, Harrow, Middlesex ; and
Walter Moss, Solar Physics Observatory, Cambridge.

Sixty presents were announced as having been received since the last meeting, including :—

International Council of Scientific Unions : Reports of Proceedings of the Second General Assembly held at Brussels, 1934 July 9 to 13.
N. E. Nörlund : Investigations of the Accuracy of relative Gravity-measurements with the Holweck-Lejay Pendulum [*in German*].
The Physical Society : International Conference on Physics, London, 1934. Three volumes. A. "Papers and Discussions" (two volumes, I. "Nuclear Physics," II. "The Solid State of Matter"); B. "Reports on Symbols, Units and Nomenclature."
The Physical Society : Reports on Progress in Physics, vol. 1.

STELLAR KINEMATICS AND THE *K*-EFFECT.

E. A. Milne.

Introduction

1. The general kinematic theory of the universe developed in a recent work * predicts that the average galaxy or sub-system should be in a state of expansion or internal radial motion from the nucleus outwards. The expansion predicted was of a totally different character from that of the expansion of the universe as a whole, the rate of the former depending on a certain function $\psi(\xi)$ describing the distribution of matter and motion in the universe, whilst the rate of the latter is independent of $\psi(\xi)$. Formulæ were given for the average distribution in particle-density in a galaxy, and for the position and velocity of any particle (a member of a given galaxy) at any given time, reckoned by the observer at the nucleus. These formulæ were developed from deep-seated kinematic considerations, without recourse to the assumption of the existence of "laws of nature" or appeal to any specific theory of gravitation, but the results were shown to be capable of a simple dynamical interpretation on an action-at-a-distance view of gravitation. For, on such a view, each particle may be considered as acted on by the gravitational pull of the whole universe, and accordingly subject to an acceleration towards the apparent centre of symmetry of the whole universe in the frame in which the particle in question is momentarily at rest ; and this "centre," for any chosen particle at any chosen epoch, was shown to be situated, on the average, on the further side of the particle, away from the nucleus to which it technically "belongs." Apart from

* *Relativity, Gravitation and World-Structure*, 1935 ; *v.* pp. 198, 279.

interlopers from other galaxies, the particles in the vicinity of a given galactic nucleus should therefore exhibit statistically a predominant outward motion. The formulæ derived were adequate to relate the density-distribution in a sub-system to the expansion explicitly, but much work remains to be done before simple quantitative laws can be predicted.

2. The observed forms of the spiral nebulæ have suggested to many that each system is in a state of radial expansion. Such a motion, combined with a rotation, has indeed been claimed to have been established by van Maanen.* Various suggestions have been put forward as to the possible origin of such a motion. H. Vogt in a series of papers has investigated † the effect of the hypothetical "cosmical repulsion" (invoked to explain the recession of the nebulæ) in determining the spiral forms of the arms of nebulæ; his conclusions have been criticised by Hacker.‡ Vogt's proposed explanation, though depending on a view of gravitation differing fundamentally from my own, is akin to the explanation to which I have been led in that it attributes the outward motion to the gravitational field of the whole universe. A different explanation has been proposed by Pilowski,§ who considered the outward motion a consequence of the supposed secular loss of mass of a galaxy.

If the expansion exists it may be called the "galactic *K*-effect." It is to be clearly distinguished from the *local* expansion shown by the average, usually positive, radial velocity of the more distant stars of our own neighbourhood. The latter may be called the "differential *K*-effect"; it is what is commonly called the "*K*-effect" without qualification.

The differential *K*-effect has been examined in detail by Pilowski in the above-mentioned papers, by von der Pahlen and Freundlich || and by Rosenhagen,¶ amongst others. Von der Pahlen and Freundlich attributed it to the motion of a cluster of stars moving as a whole about a centre of attraction, and showed that their proposed explanation also predicted a "double wave" in longitude in the radial velocities.

3. This "double wave" in radial velocity had, however, been previously predicted by Oort ** on the assumption of the existence of differential galactic rotation. Oort was strongly influenced by the investigations of Lindblad †† concerning the structure of the galaxy as a superposition of rotating systems possessing a common axis of rotation. As is well known, Oort showed that the amplitude of the double wave in the radial velocities

* *Cf.* Hubble, *Astrophys. Journ.*, **81**, 334, 1935; van Maanen, *ibid.*, **81**, 336, 1935.

† *A.N.*, **241**, Nr. 5773, 1931; **242**, Nr. 5793, 1931; **243**, Nr. 5832, 1931; **245**, Nr. 5873, 1932, etc.

‡ *Astronom. Journ.*, 977, 1932.

§ *Zeits. f. Astrophys.*, **3**, 53, 1931; **3**, 279, 1931; **3**, 291, 1931; **4**, 396, 1932.

|| *Pub. Astrophys. Obs. Potsdam*, **26**, Nr. 86, 1928.

¶ *A.N.*, **242**, Nr. 5807–8, S. 400, 1931.

** *B.A.N.* (No. 120), **3**, 275, 1927; (No. 132) **4**, 79, 1927; (No. 159), **4**, 269, 1928.

†† See, in particular, "Die Milchstrasse," *Handbuch d. Astrophys.*, Bd. 6, 1933. Also his article in *Scientia*, 1932 May.

should be proportional to the mean distance of the stars concerned. The directions of the maxima and minima or zeros in residual radial velocities, on the hypothesis of galactic rotation, determine the direction of the galactic centre. Pilowski showed, however, that the superposition of galactic expansion on galactic rotation would cause a rotation of the axes of zero residual radial velocity, whilst leaving the double wave in existence.

4. Thus it appears that various dynamical hypotheses for the origin of galactic rotation, galactic expansion or other motion-structure agree in predicting similar differential effects in radial velocities, though the details have different interpretations on the different hypotheses. The origin of this agreement was clearly indicated by Pilowski,* who emphasised that the differential radial-velocity and proper-motion behaviours were simply kinematic effects : "The use of a secular decrease of mass is only a formal intermediary for analysing the purely kinematic significance of the observed phenomena, whose physical basis remains open." The importance of Pilowski's work seems to me to be not in his suggested explanation of the outward motions inside galaxies, but in his recognition that the effects are kinematic consequences compatible with a variety of theories.

5. It is clear that a pre-requisite to a specific *dynamical* theory of the galaxy is a kinematic analysis of the observational behaviour to be expected on *any* theory. We want to know what effects will be common to all theories, inevitable on any theory, and what effects are special to a particular theory. Those effects which are shown to be inevitable cannot be taken as confirming any particular theory.

The following investigation, prompted by my own proposed explanation of the K-effect, is a purely kinematic analysis. It agrees with Pilowski's results, though following a different algebraic method. It will be shown that any spatio-velocity stellar distribution-function whatever which possesses certain differential coefficients is accompanied by, in general, a K-term and a double wave in radial velocities, and a systematic drift and an associated double wave in the transverse motions, in any arbitrary plane. The result is capable of immediate specialisation to the particular cases of galactic rotation and galactic expansion. Instead, however, of proceeding by modifying the analyses of Oort and his successors as they stand, I here apply to stellar motions the method first used by Stokes,† in a classical memoir of 1845, for discussing differential displacements in a strained elastic solid and differential motions in a (possibly viscous and compressible) fluid. Stokes showed that any displacement of an elastic solid, measured relative to a given particle of the solid in the immediate neighbourhood of that particle, could be resolved into a dilatation (positive or negative), a shear and a rotation ; and that any motion of a fluid, measured relative to a given particle of the fluid in the immediate neighbourhood of that particle, could be resolved into a rate of dilatation, a rate of shear and a spin (the shear

* *Loc. cit.*, p. 279.

† Sir G. G. Stokes, "On Theories of the Internal Friction of Fluids in Motion and of the Equilibrium and Motion of Elastic Bodies," *Trans. Camb. Phil. Soc.*, 8, 287, 1845 ; *Math. Phys. Papers*, 1, 75, 1880.

or rate of shear were describable by a strain—or rate-of-strain—ellipsoid). The importance of Stokes's work was that it laid the foundations of the present macroscopic theories of elasticity and viscosity, independent of any dynamical or molecular theory of their origins.

Oort's analysis is in effect Stokes's method applied to the particular case of galactic rotation. It thus appears that the existence of a double wave in radial velocities is *by itself* no evidence for galactic rotation, though this is not to deny the reality of galactic rotation on other evidence.*

A by-product of the investigation is that it is shown that when applying corrections for "solar motion," in order that the results may have simple physical interpretations, it is necessary to use the solar motion as given by the stars *immediately* surrounding the Sun, and not that derived from a reduction of the motions of the stars under consideration.

Definitions

6. Consider a system of stars in motion of any kind whatever. Choose any Galilean † space-frame of reference, and in that frame choose an origin O fixed relative to the frame. Later we shall choose O to be the centre of the galaxy, but for the present the choice of O is unrestricted.

7. Let P be any point in the frame, of co-ordinates X, Y, Z with respect to O, and let U, V, W be the components of velocity, in the frame, of a star at P. Let

$$f(X, Y, Z, t, U, V, W)dXdYdZdUdVdW$$

be the number of stars inside the volume element $dXdYdZ$ at P at time t with velocities lying inside the velocity-element $dUdVdW$ centred on U, V, W. Then the density n of stars at X, Y, Z at time t is given by

$$n = n(X, Y, Z, t) = \iiint f(X, Y, Z, t, U, V, W)dUdVdW, \qquad (1)$$

and the mean velocity of the stars near (X, Y, Z) at time t has components $\bar{U}, \bar{V}, \bar{W}$, where

$$\bar{U} = \bar{U}(X, Y, Z, t) = \frac{1}{n}\iiint f(X, Y, Z, t, U, V, W)UdUdVdW, \qquad (2)$$

etc.

8. Now let P_0 be the Sun, *i.e.* the star from which observations are made. Let its co-ordinates be X_0, Y_0, Z_0, its velocity in the frame U_0, V_0, W_0. Then the density at P_0 at time t is n_0 or $n(X_0, Y_0, Z_0, t)$, and the mean velocity at t at P_0 is

$$\bar{U}(X_0, Y_0, Z_0, t), \qquad \bar{V}(X_0, Y_0, Z_0, t), \qquad \bar{W}(X_0, Y_0, Z_0, t)$$

or, say,

$$\bar{U}_0, \qquad \bar{V}_0, \qquad \bar{W}_0.$$

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* *E.g.*, from the systematic motions of the globular clusters.

† Newtonian relativity is adequate for the purposes of the present paper.

Local Solar Motion

9. The components of the velocity of the Sun P_0 relative to its *immediate* surroundings are

$$U_0 - \bar{U}_0, \qquad V_0 - \bar{V}_0, \qquad W_0 - \bar{W}_0.$$

This velocity will be called the "local solar motion" and denoted by S_1^0, S_2^0, S_3^0. This phrase "local solar motion" thus denotes something different from the "solar motion" as usually used. The "solar motion" with respect to the stars at (X, Y, Z) at t, for example, is, say, $S_1(X, Y, Z, t)$, $S_2(X, Y, Z, t)$, $S_3(X, Y, Z, t)$, where

$$S_1 = U_0 - \bar{U}(X, Y, Z, t),$$

etc. This differs by terms of order $X - X_0$, $Y - Y_0$, $Z - Z_0$ from the local solar motion, and it is precisely the effect of differences of this order which we are now about to discuss.

Radial Velocities observed from P_0

10. Let x, y, z be the co-ordinates of P relative to the Sun, so that

$$x = X - X_0, \qquad y = Y - Y_0, \qquad z = Z - Z_0. \tag{3}$$

Let r be the distance P_0P. Let u, v, w be the components of velocity of a star at P relative to P_0, so that

$$u = U - U_0, \qquad v = V - V_0, \qquad w = W - W_0. \tag{4}$$

Then the observed radial velocity of the star, as seen from the Sun, is, say, $\rho(x, y, z, t)$, where

$$\begin{aligned}\rho(x, y, z, t) &= u\frac{x}{r} + v\frac{y}{r} + w\frac{z}{r} \\ &= (U - U_0)\frac{x}{r} + (V - V_0)\frac{y}{r} + (W - W_0)\frac{z}{r}.\end{aligned}$$

The mean radial velocity * $\bar{\rho}(x, y, z)$ of the stars at x, y, z at time t is given by

$$\bar{\rho}(x, y, z) = \frac{1}{n}\iiint\left[(U - U_0)\frac{x}{r} + (V - V_0)\frac{y}{r} + (W - W_0)\frac{z}{r}\right] f(X, Y, Z, t, U, V, W)\,dU\,dV\,dW$$

$$= (\bar{U} - U_0)\frac{x}{r} + (\bar{V} - V_0)\frac{y}{r} + (\bar{W} - W_0)\frac{z}{r}. \tag{5}$$

But

$$\begin{aligned}\bar{U} - U_0 &= (\bar{U} - \bar{U}_0) - (U_0 - \bar{U}_0) \\ &= \bar{U}(X, Y, Z) - \bar{U}(X_0, Y_0, Z_0) - S_1^0.\end{aligned}$$

If P is not too far from P_0, this gives as a first approximation

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* It is convenient henceforth to omit the variable t, although a steady state is not assumed.

$$\bar{U} - U_0 = x\left(\frac{\partial \bar{U}}{\partial X}\right)_0 + y\left(\frac{\partial \bar{U}}{\partial Y}\right)_0 + z\left(\frac{\partial \bar{U}}{\partial Z}\right)_0 - S_1{}^0, \tag{6}$$

where the suffix o implies that the differential coefficients concerned are to be evaluated for $X = X_0$, $Y = Y_0$, $Z = Z_0$ at time t. Substituting from (6) and its analogues in (5) we find

$$\bar{\rho}(x, y, z) + \sum S_1{}^0 \frac{x}{r} = \frac{1}{r}[Ax^2 + By^2 + Cz^2 + 2Fyz + 2Gzx + 2Hxy], \tag{7}$$

where

$$A = \left(\frac{\partial \bar{U}}{\partial X}\right)_0, \qquad B = \left(\frac{\partial \bar{V}}{\partial Y}\right)_0, \qquad C = \left(\frac{\partial \bar{W}}{\partial Z}\right)_0,$$

$$F = \tfrac{1}{2}\left[\frac{\partial \bar{V}}{\partial Z} + \frac{\partial \bar{W}}{\partial Y}\right]_0, \quad G = \tfrac{1}{2}\left[\frac{\partial \bar{W}}{\partial X} + \frac{\partial \bar{U}}{\partial Z}\right]_0, \quad H = \tfrac{1}{2}\left[\frac{\partial \bar{U}}{\partial Y} + \frac{\partial \bar{V}}{\partial X}\right]_0.$$

The array

$$\begin{matrix} A, & H, & G, \\ H, & B, & F, \\ G, & F, & C, \end{matrix}$$

gives the components of the self-conjugate "mean rate-of-strain tensor," which will occasionally be denoted by $\mathbf{T}$.

11. The left-hand side of (7) is the mean radial velocity of the stars at P, or (x, y, z), as viewed from P_0, corrected for *local* solar motion. It will be denoted by $\bar{\rho}'(x, y, z)$.

The right-hand side of (7), for stars in a fixed direction, vanishes as $r \to 0$, and $\bar{\rho}(x, y, z)$ for *very* near stars is accordingly a function of x, y, z given by

$$\bar{\rho}(x, y, z) = -\left[S_1{}^0 \frac{x}{r} + S_2{}^0 \frac{y}{r} + S_3{}^0 \frac{z}{r}\right].$$

The methods of deriving $\mathbf{S}^0$ from this property are well known. We suppose $\mathbf{S}^0$ so determined and the correction for local solar motion applied, and we have then

$$\bar{\rho}'(x, y, z) = \frac{1}{r}[Ax^2 + By^2 + Cz^2 + 2Fyz + 2Gzx + 2Hxy] \tag{7'}$$

$$= \frac{1}{r}\mathbf{T} : \mathbf{rr}.$$

The right-hand side * is the quotient of a quadratic form in x, y, z by r, and the coefficients are functions of X_0, Y_0, Z_0, t only.

12. Consider the behaviour of this quadratic. From the fundamental theorem on the reduction of a quadratic form to its principal axes it is known that for fixed $r(= |\mathbf{r}|)$, $\mathbf{T} : \mathbf{rr}$ takes its algebraically greatest and least values when $\mathbf{r}$ lies along two of the three axes of the tensor quadric of $\mathbf{T}$,

* The colon (:) denotes the double inner product of the tensor $\mathbf{T}$ with the dyad $\mathbf{rr}$, *i.e.* $T_{\mu\nu} r_\nu r_\mu$ in suffix notation.

namely, $\mathbf{T} : \mathbf{rr} = \text{const.}$ It follows that if we examine the mean radial velocity corrected for local solar motion, $\bar{\rho}'(x, y, z)$, for stars at a given distance r from P_0 in various directions, then we shall find two perpendicular directions in which it takes its maximum and minimum values respectively.

13. Let us average $\bar{\rho}'$ for stars at distance r over the whole sky, averaging not according to the number of stars in each element of solid angle, but according to the elements of solid angle themselves. Denote the result by $\bar{\bar{\rho}}'(r)$. Then *

$$\bar{\bar{\rho}}'(r) = \frac{1}{r}\iint (Ax^2 + By^2 + Cz^2 + 2Fyz + 2Gzx + 2Hxy)\frac{d\omega}{4\pi}$$
$$= \tfrac{1}{3}r(A + B + C). \tag{8}$$

This is the "K-term" at distance r to a first approximation. We see † that it is proportional to r. We see at once why the differential K-effect only shows up for the more distant stars, such as the B-stars. We see also that when stellar motions are analysed for a K-term, the K-term must be taken not as a constant, but as proportional to the distance. Lastly, we see that any distribution of stellar velocities will in general show a K-effect.‡ It is convenient to write

$$K(r) = \tfrac{1}{3}r(A + B + C) = \tfrac{1}{3}r\left[\frac{\partial \bar{U}}{\partial X} + \frac{\partial \bar{V}}{\partial Y} + \frac{\partial \bar{W}}{\partial Z}\right]_0. \tag{9}$$

14. Removing the K-term, we have now

$$\bar{\rho}'(x, y, z) - K(r) = \frac{1}{r}\left[\sum \tfrac{1}{3}x^2(2A - B - C) + 2\sum Fyz\right]. \tag{10}$$

It is known from the theory of quadratic forms that $A + B + C$ is an invariant, independent of the axes chosen; this follows also since $A + B + C$ is the divergence of the velocity $(\bar{U}, \bar{V}, \bar{W})$ at P_0. If we choose axes parallel to the principal axes of the rate-of-strain tensor, the general quadratic expression will reduce to

$$A'x^2 + B'y^2 + C'z^2,$$

where

$$A' + B' + C' = A + B + C,$$

and referred to these axes we have for the residuals

$$\bar{\rho}'(x, y, z) - K(r) = \frac{1}{r}[\tfrac{1}{3}x^2(2A' - B' - C') + \tfrac{1}{3}y^2(2B' - C' - A') + \tfrac{1}{3}z^2(2C' - A' - B')]. \tag{10'}$$

If $A' > B' > C'$, the maximum value of the mean residual radial velocity, corrected for local solar motion and for K-effect, for stars at distance r in different directions, is $\frac{1}{3}r(2A' - B' - C')$, and the minimum is $-\frac{1}{3}r(A' + B' - 2C')$. These occur in perpendicular directions.

15. Instead of referring the rate-of-strain tensor to its principal axes

* The integration is immediate on using local spherical polar co-ordinates.

† The algebraic sign may be positive or negative *a priori*.

‡ Result due to Pilowski.

we may continue to leave the axes arbitrary and examine the behaviour of the mean radial velocity (corrected for local solar motion) in any arbitrary plane. Choose this plane to be $z=0$. Then in this plane

$$\bar{\rho}'(x, y)=\frac{1}{r}(Ax^2+2Hxy+By^2). \tag{11}$$

In this plane, take a longitude variable λ and put

$$x=r\cos\lambda, \qquad y=r\sin\lambda.$$

Then

$$\bar{\rho}'(r, \lambda)=r[\tfrac{1}{2}(A+B)+\tfrac{1}{2}(A-B)\cos 2\lambda+H\sin 2\lambda]. \tag{12}$$

The maximum and minimum values of $\bar{\rho}'(r, \lambda)$, for fixed r, occur where

$$\tan 2\lambda=\frac{2H}{A-B}.$$

If the solutions of this are

$$2\lambda=2\lambda_0, \qquad 2\lambda_0+\pi, \qquad 2\lambda_0+2\pi, \qquad 2\lambda_0+3\pi,$$

the directions of the maximum and minimum of the mean radial velocity are given by

$$\lambda=\lambda_0, \qquad \lambda_0+\tfrac{1}{2}\pi, \qquad \lambda_0+\pi, \qquad \lambda_0+\tfrac{3}{2}\pi.$$

Since maxima and minima must alternate, the directions of maxima are perpendicular to the directions of minima in the chosen plane.

Let the axes of x, y in the chosen plane be rotated so that

$$Ax^2+2Hxy+By^2$$

becomes

$$A'x'^2+2H'x'y'+A'y'^2,$$

where in the new form the coefficients of x'^2 and y'^2 are equal. This is clearly possible. Then put $x'=r\cos\lambda'$, $y'=r\sin\lambda'$, where λ' is the longitude measured from the direction $\lambda_0-\frac{1}{4}\pi$. Then we have

$$\bar{\rho}'(r, \lambda')=r(A'+H'\sin 2\lambda'), \tag{13}$$

where

$$A'=\tfrac{1}{2}(A+B)=\left(\frac{\partial\bar{U}'}{\partial X'}\right)_0=\left(\frac{\partial\bar{V}'}{\partial Y'}\right)_0, \tag{14}$$

$$H'=[H^2+\tfrac{1}{4}(A-B)^2]^{\frac{1}{2}}=\tfrac{1}{2}\left[\frac{\partial\bar{U}'}{\partial Y'}+\frac{\partial\bar{V}'}{\partial X'}\right]_0. \tag{15}$$

Formula (13) shows that in the chosen plane, round the circle r, there is a mean K-term rA', and superposed on this a double wave $rH'\sin 2\lambda'$. We recollect that we have made no hypothesis of galactic rotation, or any other specialisation of the mean velocity as a function of position. Further, the plane is arbitrary. Thus, for any arbitrary spatio-velocity distribution function which gives differentiable components for the mean velocity, in any arbitrary plane, a K-term and superposed double wave in longitude are

exhibited by the mean radial velocities corrected for local solar motion. It follows, as stated in the introduction, that the observational verification of the existence of a K-term and a double wave in longitude, in any plane, is by itself not evidence in favour of any particular dynamical interpretation.

A word may be said as to the accuracy of (13). For r very small the mean corrected radial velocity in any direction is also small; the correction for local solar motion is in fact determinable by treating it as zero. As stars of greater distance are examined a K-term and a second harmonic in longitude make their appearance, of sizes increasing proportionally to the distance. Later on, if still more distant stars are examined, terms proportional to higher powers of the distance (here neglected) will appear, accompanied also by higher harmonics.

16. Later we shall specialise the foregoing formulæ by using galactic co-ordinates. Before doing this it is convenient to analyse the mean transverse proper motions in the same general fashion.

Transverse Velocities

17. We keep the directions of the axes arbitrary as before, but specify them to form a right-handed set. The component of the transverse velocity at P, observed from P_0, parallel to the plane P_0xy, is easily seen to be

$$\frac{-uy+vx}{(x^2+y^2)^{\frac{1}{2}}}$$

from right to left. Call this $\rho_e(x, y, z)$, the suffix e standing for "equatorial."

The component of the transverse velocity perpendicular to the above direction (*i.e.* in a direction intersecting the z-axis) is seen to be

$$\frac{-uzx-vzy+w(x^2+y^2)}{r(x^2+y^2)^{\frac{1}{2}}},$$

positive towards the positive side of the xy-plane. Call this $\rho_p(x, y, z)$, the suffix p standing for "polar."

Taking the averages at (x, y, z) and correcting for local solar motion, we find by the same procedure as before

$$\bar{\rho}_e'(x, y, z) = (x^2+y^2)^{-\frac{1}{2}}\Big[-y\Big\{x\frac{\partial\bar{U}}{\partial X}+y\frac{\partial\bar{U}}{\partial Y}+z\frac{\partial\bar{U}}{\partial Z}\Big\}_0 + x\Big\{x\frac{\partial\bar{V}}{\partial X}+y\frac{\partial\bar{V}}{\partial Y}+z\frac{\partial\bar{V}}{\partial Z}\Big\}_0\Big], \quad (16)$$

$$\bar{\rho}_p'(x, y, z) = \frac{(x^2+y^2)^{-\frac{1}{2}}}{r}\Big[-zx\Big\{x\frac{\partial\bar{U}}{\partial X}+y\frac{\partial\bar{U}}{\partial Y}+z\frac{\partial\bar{U}}{\partial Z}\Big\}_0 - zy\Big\{x\frac{\partial\bar{V}}{\partial X}+y\frac{\partial\bar{V}}{\partial Y}+z\frac{\partial\bar{V}}{\partial Z}\Big\}_0 + (x^2+y^2)\Big\{x\frac{\partial\bar{W}}{\partial X}+y\frac{\partial\bar{W}}{\partial Y}+z\frac{\partial\bar{W}}{\partial Z}\Big\}_0\Big]. \quad (17)$$

(The angular proper motions follow on division by r.)

These can be discussed in the same way as the radial velocities. Here we

shall confine attention to the mean equatorial component, for stars in the arbitrary plane $z=0$. Equation (16) yields

$$\bar{\rho}_e'(r,\lambda)=r\left[\tfrac{1}{2}\left(\frac{\partial\bar{V}}{\partial X}-\frac{\partial\bar{U}}{\partial Y}\right)_0+\tfrac{1}{2}\left(\frac{\partial\bar{V}}{\partial X}+\frac{\partial\bar{U}}{\partial Y}\right)_0\cos 2\lambda+\tfrac{1}{2}\left(\frac{\partial\bar{V}}{\partial Y}-\frac{\partial\bar{U}}{\partial X}\right)\sin 2\lambda\right]$$

$$=r[\omega_3+H\cos 2\lambda+\tfrac{1}{2}(B-A)\sin 2\lambda], \quad (18)$$

where A, B, H are as before and ω_3 is the z-component of half the *curl* of $(\bar{U}, \bar{V}, \bar{W})$, *i.e.* the local spin. This may be compared with (12). It will be seen that there is a systematic drift round the z-axis, and a double wave in longitude of the same amplitude as that in the radial velocities. We see again that these features are not special to galactic rotation, but hold good for any distribution of velocities in any arbitrary plane.

Specialisation for Galactic Symmetry

18. We shall now assume that the galaxy has a plane of symmetry $Z=0$, and we shall choose the origin O to be the centre of the galaxy. We shall not, however, assume that the galaxy is azimuthally symmetrical about the axis of Z, for the analogy with spiral nebulæ suggests the possibility that this may not be so. Let R, θ, Z be cylindrical polar co-ordinates with respect to O, θ being measured from the prolongation of OP_0; and let l, b be *local* galactic longitude and latitude measured from P_0, so that

$$x=r\cos b\cos l, \qquad y=r\cos b\sin l, \qquad z=r\sin b.$$

The longitude l is measured from the prolongation of OP_0, positive in the anti-clockwise sense as seen from the side on which Z is positive.

Let L, M be the velocity components * of an object at P, in the direction along R and perpendicular to R in the sense of increasing θ. Simple transformations of co-ordinates and vectors then gives for the velocity-gradients at P_0 the following formulæ :—

$$\left(\frac{\partial\bar{U}}{\partial X}\right)_{\theta=0}=\frac{\partial\bar{L}}{\partial R}, \qquad \left(\frac{\partial\bar{U}}{\partial Y}\right)_{\theta=0}=\frac{1}{R}\frac{\partial\bar{L}}{\partial\theta}-\frac{\bar{M}}{R},$$

$$\left(\frac{\partial\bar{V}}{\partial X}\right)_{\theta=0}=\frac{\partial\bar{M}}{\partial R}, \qquad \left(\frac{\partial\bar{V}}{\partial Y}\right)_{\theta=0}=\frac{1}{R}\frac{\partial\bar{M}}{\partial\theta}+\frac{\bar{L}}{R}.$$

We shall now make the usual assumptions † that the mean motion normal to the galactic plane is zero, $\bar{W}=0$, and that $\bar{U}$, $\bar{V}$ (or $\bar{L}$, $\bar{M}$) are independent of Z. Then $C=F=G=0$. Our formulæ (7′), (16), (17) then yield

$$\bar{\rho}'(r,b,l)=r\cos^2 b[\tfrac{1}{2}(A+B)+\tfrac{1}{2}(A-B)\cos 2l+H\sin 2l], \quad (19)$$

---

* L, M are in Lindblad's notation Π, Θ. They are to be distinguished from $\bar{L}$ and $\bar{M}$. Lindblad's notation appears to derive from Jeans' paper, *M.N.*, **82**, 122, 1922.

† It would be equally possible to write down formulæ expressing the existence of an outward mean velocity directed from the centre; in that case $\bar{W}$ would not be zero. Such developments have been considered by Pilowski.

$$\bar{\rho}_e'(r, b, l) = r \cos b[\omega_3 + H \cos 2l - \tfrac{1}{2}(A - B) \sin 2l], \tag{20}$$

$$\bar{\rho}_p'(r, b, l) = -r \cos b \sin b[\tfrac{1}{2}(A + B) + \tfrac{1}{2}(A - B) \cos 2l + H \sin 2l]. \tag{21}$$

(The transverse polar component is seen to be simply $-\tan b$ times the radial component.)

In these formulæ the constants are given by

$$\tfrac{1}{2}(A + B) = \tfrac{1}{2}\left[\frac{\partial \bar{L}}{\partial R} + \frac{\bar{L}}{R} + \frac{1}{R}\frac{\partial \bar{M}}{\partial \theta}\right]_0, \tag{22}$$

$$\tfrac{1}{2}(A - B) = \tfrac{1}{2}\left[\frac{\partial \bar{L}}{\partial R} - \frac{\bar{L}}{R} - \frac{1}{R}\frac{\partial \bar{M}}{\partial \theta}\right]_0, \tag{23}$$

$$H = \tfrac{1}{2}\left[\frac{\partial \bar{M}}{\partial R} - \frac{\bar{M}}{R} + \frac{1}{R}\frac{\partial \bar{L}}{\partial \theta}\right]_0, \tag{24}$$

$$\omega_3 = \tfrac{1}{2}\left[\frac{\partial \bar{M}}{\partial R} + \frac{\bar{M}}{R} - \frac{1}{R}\frac{\partial \bar{L}}{\partial \theta}\right]_0. \tag{25}$$

19. The following particular cases are of great interest :—

(*a*) *Pure Galactic Rotation.*—$\bar{L} = 0$, $\bar{M}$ independent of θ. Then

$$\tfrac{1}{2}(A + B) = 0, \qquad \tfrac{1}{2}(A - B) = 0, \qquad H = \tfrac{1}{2}\left(\frac{\partial \bar{M}}{\partial R} - \frac{\bar{M}}{R}\right)_0, \qquad \omega_3 = \tfrac{1}{2}\left(\frac{\partial \bar{M}}{\partial R} + \frac{\bar{M}}{R}\right)_0.$$

Equations (19) and (20) reduce to

$$\bar{\rho}'(r, b, l) = r\tfrac{1}{2}\left(\frac{\partial \bar{M}}{\partial R} - \frac{\bar{M}}{R}\right)_0 \cos^2 b \sin 2l, \tag{19'}$$

$$\bar{\rho}_e'(r, b, l) = r \cos b\left[\tfrac{1}{2}\left(\frac{\partial \bar{M}}{\partial R} + \frac{\bar{M}}{R}\right)_0 + \tfrac{1}{2}\left(\frac{\partial \bar{M}}{\partial R} - \frac{\bar{M}}{R}\right)_0 \cos 2l\right]. \tag{20'}$$

These are Oort's formulæ, and they have the same order of accuracy as his. If we write $\bar{M} = R\Omega$, where Ω is the angular velocity of the galaxy about its centre at distance R, then

$$H = \tfrac{1}{2}\left(\frac{\partial \bar{M}}{\partial R} - \frac{\bar{M}}{R}\right) = \tfrac{1}{2}R\frac{d\Omega}{dR},$$

$$\omega_3 = \tfrac{1}{2}\left(\frac{\partial \bar{M}}{\partial R} + \frac{\bar{M}}{R}\right) = \Omega + \tfrac{1}{2}R\frac{d\Omega}{dR},$$

and

$$\Omega = \omega_3 - H.$$

(*b*) *Pure Galactic Expansion parallel to the Galactic Plane.*—$\bar{M} = 0$, $\bar{L}$ independent of θ. Then

$$\tfrac{1}{2}(A + B) = \tfrac{1}{2}\left(\frac{\partial \bar{L}}{\partial R} + \frac{\bar{L}}{R}\right)_0, \qquad \tfrac{1}{2}(A - B) = \tfrac{1}{2}\left(\frac{\partial \bar{L}}{\partial R} - \frac{\bar{L}}{R}\right), \qquad H = 0, \qquad \omega_3 = 0.$$

Equations (19) and (20) reduce to

$$\bar{\rho}'(r, b, l) = r \cos^2 b\left[\tfrac{1}{2}\left(\frac{\partial \bar{L}}{\partial R} + \frac{\bar{L}}{R}\right)_0 + \tfrac{1}{2}\left(\frac{\partial \bar{L}}{\partial R} - \frac{\bar{L}}{R}\right)_0 \cos 2l\right], \qquad (19'')$$

$$\bar{\rho}_e'(r, b, l) = -r\tfrac{1}{2}\left(\frac{\partial \bar{L}}{\partial R} - \frac{\bar{L}}{R}\right)_0 \cos b \sin 2l. \qquad (20'')$$

20. If in (19″) and (20″) we rotate the zero of l through $-\frac{1}{4}\pi$, by writing $l' = l + \frac{1}{4}\pi$, we have to replace $\cos 2l$ by $\sin 2l'$ and $-\sin 2l$ by $\cos 2l'$. Hence, unless we already know the direction of the galactic centre, $l = \pi$, the harmonic terms by themselves do not serve to distinguish galactic rotation from galactic expansion. On the other hand, the occurrence of a constant term (proportional to r) in the radial velocities would show that the formula (19′) for pure galactic rotation would not fit the facts, and hence $\bar{L}$ could not be zero ; this would accordingly be evidence for galactic expansion. Similarly the occurrence of a constant term (proportional to r) in the equatorial transverse motions would show that the formula (20″) for pure galactic expansion would not fit the facts, and hence $\bar{M}$ could not be zero ; this would accordingly be evidence for galactic rotation. Since both terms are known to be present, we conclude that the galaxy is both rotating and expanding.

It should be noticed that though it is *necessary*, if galactic expansion is absent, for the constant term in (19) to be zero, this condition by itself would not be *sufficient* to establish the absence of galactic expansion, assuming $\bar{M}$ independent of θ. For we must have also $A - B = 0$, *i.e.* the zero of the harmonic term in (19) must agree with the direction of the galactic centre. Absence of K-term gives simply $A + B = 0$, *i.e.* $\frac{1}{2}(A - B) = A = (\partial \bar{L}/\partial R)_0$, and unless this is also zero $\bar{L}$ will not be zero.

21. Comparison of (19′) with (19) shows that the effect of superposing galactic expansion on pure galactic rotation, besides introducing a K-term, is *in general* to rotate the directions of zero residual radial velocity away from the direction to and from, and perpendicular to, the galactic centre. This is Pilowski's result. Any discrepancy between the true direction of the galactic centre, as determined by star-counts, and the direction of one of the zeros in residual radial velocity may be attributed to an effect of differential galactic expansion. Let us write

$$\tfrac{1}{2}(B - A) = \sigma \sin 2l_0, \qquad H = \sigma \cos 2l_0,$$

where

$$\sigma = +[H^2 + \tfrac{1}{4}(B - A)^2]^{\frac{1}{2}},$$

and also put

$$k = \tfrac{1}{2}(A + B) ;$$

then we find from (19), (20) the relations

$$\bar{\rho}'(r, b, l) = r \cos^2 b[k + \sigma \sin 2(l - l_0)], \qquad (26)$$

$$\bar{\rho}_e'(r, b, l) = r \cos b[\omega_3 + \sigma \cos 2(l - l_0)]. \qquad (27)$$

If the direction of the galactic centre ($l = \pi$) is supposed known from other considerations, the graphs of the radial and equatorial transverse velocities

will determine : (i) the constant terms rk and $r\omega_3$; (ii) the amplitude $r\sigma$, which should be the same for the two curves ; (iii) the value of $2l_0$. Actually in (27) *angular* proper motions will be used, so that the ratio of the amplitude of the harmonic term in the radial velocities to that of the harmonic term in the equatorial proper motions gives r directly. Hence k, ω_3 and σ can be found. Assuming the θ-derivatives in (22) to (25) to be zero, we then find explicitly

$$\Omega = \frac{\bar{M}}{R} = \omega_3 - \sigma \cos 2l_0, \qquad \frac{\partial \bar{M}}{\partial R} = \omega_3 + \sigma \cos 2l_0. \tag{28}$$

$$\frac{\bar{L}}{R} = k + \sigma \sin 2l_0, \qquad \frac{\partial \bar{L}}{\partial R} = k - \sigma \sin 2l_0. \tag{29}$$

These formulæ give at once the rotation and the expansion (positive or negative), with their gradients, in terms of observational data. It is particularly to be noted that a result $l_0 = 0$ would not imply $\bar{L} = 0$ unless also $k = 0$; $l_0 = 0$ implies (locally) $\bar{L} = kR$; whilst $k = 0$ implies (locally) $\bar{L} \propto R^{-1}$.

22. Though it is not the intention of this paper to discuss the observational data, it may be mentioned that our results (in essential Pilowski's) may explain the anomalies disclosed in Redman's work * on the radial velocities of the late-type stars. Redman was unable to reconcile the observed radial velocities of the K-type stars with the hypothesis of pure galactic rotation ; he found, in particular, a high value for the declination of the Sun's apex, and a much higher value for his "l_0" † than the usually adopted longitude of the centre of the galaxy. His high value for "l_0" suggests the presence of differential galactic expansion. It may be mentioned that Redman solved for the solar motion from the stars concerned instead of correcting for *local* solar motion (*i.e.* instead of using the solar motion given by the naked-eye stars) ; this would tend to obliterate or reduce the second harmonic term which should theoretically remain when *local* solar motion is removed, and this is compatible with Redman's remark that he found very little improvement in residuals by the inclusion of the second harmonic term. He concluded that his work was evidence for the existence of something besides galactic rotation as affecting the radial velocities. He suggested star-streaming as the cause, but the results of the present paper are independent of the occurrence of star-streaming ; star-streaming expresses *dispersion* features of the velocity distribution, whilst we have only found it necessary to discuss mean values.

An alternative explanation, suggested to me privately by Professor H. H. Plaskett, is that Redman's stars may form a group not specifically related to the galactic plane, for we have seen that in any arbitrary plane any distribution of velocities will exhibit constant terms and terms in sin $2l$ and cos $2l$ of the same general character as those for galactic rotation plus expansion. This follows by comparison of (12) and (18) with (19) and (20).

* *Pub. Dom. Astrophys. Obs.*, **4** (No. 20), 325, 1930 ; **6** (No. 5), 27, 1931.

† Redman's "l_0" (longitude of the galactic centre from a certain zero) is to be distinguished from the "l_0" of § 21 above (longitude of a zero of the double wave in $\bar{\rho}'$).

23. Stokes's method can be applied to other problems in stellar motions; for example, systematic variations of solar motion or of the axes of star-streaming for groups of stars at different places. These will not be dealt with here.

Summary

24. Oort's method of analysing relative stellar velocities for differential galactic rotation is generalised for any distribution of velocities on the lines of Stokes's analysis of the differential motion of an element of a fluid. It is shown that, whatever the velocity-distribution, there will be in general a non-zero K-effect, a non-zero systematic trend in transverse motions and "double waves" in longitude in the radial velocities and transverse velocities in any arbitrary plane; all the terms being proportional to the distance.

The results are applied to a galaxy possessing a plane of symmetry with zero systematic motion perpendicular to it. It is shown, in agreement with Pilowski, that the effects of galactic rotation and galactic expansion are intermingled; the effect of galactic expansion, superposed on galactic rotation, is *in general* to rotate the directions of zero residual radial velocity away from the direction to the centre of the galaxy. The maximum amount of information that can be obtained from radial velocities and transverse proper motions as a function of longitude is discussed.

It is shown that when velocities are corrected for solar motion it is desirable to use the *local* solar motion instead of the solar motion derived from the stars in question.

I am indebted to Professor H. H. Plaskett for his interest in the foregoing investigation, without which it would not have been attempted. Though the idea of seeking for evidence as to galactic expansion arose from my work on cosmology, the immediate incentive was a lecture by Dr. J. S. Plaskett on stellar motions. I am also grateful to Dr. W. M. Smart and Dr. R. O. Redman for reading the paper in draft and making certain suggestions.

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THE PARALLAX OF NOVA HERCULIS 1934 DEDUCED FROM THE INTENSITY OF THE INTERSTELLAR K LINE.

E. G. Williams, M.A.

(Communicated by the Director, Solar Physics Observatory, Cambridge)

1. *Introduction.*—The distances of very few galactic novæ are known with any certainty. As the trigonometrical parallaxes are in general very small, the most trustworthy distances are those which have been derived in the few instances where an expanding nebular disc has been detected some

months or years after the outburst, by combining the rate of growth of this disc with the spectroscopic data on the velocity of expansion. The method is not a sure one, however, as there may be uncertainty in associating the observed disc with a particular one of several absorption velocities. In the case of N Aquilæ the hydrogen and nebulium shells gave at first somewhat conflicting results. Further, not all novæ have been observed to develop visible nebulous discs.

Distances derived by a quite independent method based on the intensity of interstellar lines should therefore be of value. In the present instance observations have been confined to the *H* and *K* lines. Data for the interstellar sodium lines will only be of use when more is known about the relation between *D*-line intensity and distance.

2. *The Observations.*—Thanks to the large displacement of the absorption lines due to the nova itself the undisplaced line is to be seen on nearly all plates of the *K* region in the Cambridge series ; twenty-four plates have been selected for intensity measures, on all of which the spectra were both well exposed and of reasonable width. The scale of the plates, all taken with the two-prism spectrograph on the Newall 25-inch refractor, is 15·0 A./mm. at the *K* line. Standardization was effected by exposing the other half of the plate from which the stellar spectrogram was cut in an auxiliary spectrograph fitted with a neutral tinted wedge. Not only were all selected spectra taken on the same emulsion (Ilford Astra II) and similarly developed and standardized, but the photographic density was that most suitable for photometry, and the trailing was uniformly good, so that the plates form a very homogeneous series. Further, all but three of the spectra have the unusually large width of 0·75 mm., which materially reduces the irregularities in the microphotometer record due to plate grain.

The intensity measures were made on tracings of 50-fold magnification. Two such tracings are shown in the plate. It was found convenient to consider the *K* line as triangular in shape and simply to measure the extreme width and the depth, *D*, expressed as percentage absorption of the background intensity at the wave-length of the line.

In Table I column 3 gives the total intensity, *I*, in units of complete absorption over 0·1 wave number.* Column 5 gives the plate weight, and the two following ones the data for interstellar *H*, when this line was measured.

Professor Stratton has very kindly measured a number of these plates for the radial velocity of the detached lines. His values, reduced to the Sun, appear in the last two columns of the table.

3. *Discussion. Influence of Blends.*—On the plates taken before January 22 the displacement and intensity of the stellar *K* line, subsequently referred to as K_S, are such that the interstellar line, K_I, lies on the wing of K_S. When this is the case the hypothetical contour of K_S has to be drawn in over K_I on the microphotometer tracing. Unfortunately the contour of K_S is usually much curved just where this needs to be done, and hence the strength of K_I is largely dependent on the judgment of the draughtsman. The

* This unit has been chosen to facilitate comparison with previously published *K*-line intensities. $I = 65$ for an equivalent width of one angstrom.

TABLE I

Date	Plate No.	*K* Line *I*	*K* Line *D*	Pl. Wt.	*H* Line *I*	*H* Line *D*	Radial Velocity *K*	Radial Velocity *H*	Remarks
							km./sec.		
1934 Dec. 18·7	*T* 210	11·4	20	2			−41		*K* blended
21·8	225						−21		*K* difficult.
21·8	226	2·7	13	2			−49		Vel. given half weight
31·7	248	4·7	24	2			−22	−31	
1935 Jan. 4·2	254	3·8	18	3					
5·2	261	4·9	16	3					
7·2	269	3·2	15	3			−36	−42	
10·3	275	3·5	13	3					
12·2	283	7·8	22	3	4·7	12 :	−23	−35	
15·3	295	4·0	13	3			−19	−23	
22·2	307	5·6	20	3	5·8	14			
28·1	323	13·6	24	3		8 :			
Feb. 7·3	337	12·5	32	2	4·2	11 :			
8·3	339						−15		
9·1	342	9·0	22	3		13			
13·2	350	9·2	25	3	4·9	13	−15	−23	
14·2	354	11·2	23	3					
17·1	361	12·4	25	3			−29		*H* is invisible
18·2	366	9·6	26	2	5·0	14			
23·1	372	9·0	21	3	3·5	10			
26·0	379	7·4	18	3			−15	+8	*H* wk. diffuse. Blend?
Mar. 8·2	395	8·8	19	3					
12·1	404	6·9	21	2	4·2	14			
23·1	420	12·3	19	2		13 :	− 9	−13	*H* diffuse
24·2	422	11·7	23	2	8·2 :	14	−24	−10	*H* diffuse. Blend with O^+?
26·1	425	11·5	23	3					*H* rather diffuse

equivalent widths listed in Table I result from treating each tracing on its own merits. It will be observed that for all plates taken before January 22, except the first one, the equivalent widths are consistently lower than the mean of those taken when the line is completely free from the stellar component and the position of the background, which is now the broad *K* emission band, is comparatively well defined. The early low values are explicable if the outline of K_S has in each case been drawn in too low. A similar phenomenon was noted by the writer * in deriving the ratio H/K for interstellar lines in B-type stars. The ratio was systematically too low when *H* fell on the steep contour of the hydrogen line $H\epsilon$ in sharp line stars.

As a check on this explanation, the mean central depth derived from the

* *Ap. J.*, 79, 294, 1934.

later plates, 22 per cent., has been assumed to be the correct one.* If this depth is applied to the earlier tracings the deduced position of the wing of K_S above the centre of K_I is in no case impossibly high. An example is to be seen in Plate 9, fig. *d*, where the lower of the two lines over K_I represents the stellar background as originally drawn, whilst the upper is necessitated if 22 per cent. of the stellar light is to be absorbed at the centre of K_I.

In the spectrum of α Cygni there are four absorption lines which are in positions to cause trouble by blending with K_I if they are also present in the displaced nova spectrum. The following wave-lengths (here corrected to I.A.) and intensities are from the South Kensington observations:—†

3935·85(3) *Fe*II 5·94 ; [*Fe*I 5·82(2)]	3938·45(3) *Mg*I 8·41(4)
3937·15(1) [*Fe*I 7·34(3)]	3939·15(3) *Fe*II 9·15(0) ; *Fe*II 8·97(−1)

Possible identifications and solar intensities follow the wave-lengths.

On December 18 Professor Stratton's measures give a mean velocity of −213 km. for the Fe^+ lines. If this is applied to the line λ 3935·94 its position coincides exactly with the measured position of the *K* line on this date, and as the other Fe^+ lines are strong this probably explains both the low radial velocity and high intensity. After falling to −176 km. on December 21 the Fe^+ velocity increases again rapidly and is already −300 km. on December 28, which means that after this date λ 3935·94 will remain clear to the violet side of K_I.

If the second identification is correct, λ 3937·34 would blend with K_I for velocities 295 ± 60 km. It is certain, however, that in the nova this *Fe*I line is absent, for none of the stronger lines in the same multiplet are visible. In fact only the strongest neutral iron lines are to be seen.

The last two of the four lines listed blend with our dispersion, but in the nova the *Mg* line will be absent, as two stronger lines of its multiplet are invisible though seen in α Cygni. I am indebted to Dr. Dobbie for the identification of the two lines λ 3935·94(4) and λ 3938·97(3). His unpublished material shows that they are members of a pair of medium level doublets in the Fe^+ spectrum, being associated respectively with the lines λ 3906·04(3) and λ 4002·55(2). The figures in brackets give the approximate intensities in the arc. The two last-mentioned lines are both distinctly seen on earlier plates and are moderately strong in early January. The plates from December 31 to January 12 show distinctly a line to the red of *K*. This line was first detected by Guthnick,‡ who reported that interstellar *K* was variable in intensity and at times appeared to be double. If the line to the red is identified with λ 3938·97 the velocity displacement is about −355 km. In order that this iron line shall coincide in position with undisplaced *K* its displacement must reach −430 km. The Fe^+ lines, such as λ 4233, do not get within blending distance of this displacement until they are fading in intensity, whilst the companion lines $\lambda\lambda$ 3906, 4002, are

* This depth is, of course, mainly decided by instrumental factors.

† Lockyer and Baxandall, *Phil. Trans.*, A, **201**, 205, 1903.

‡ *A.N.*, **254**, 159, 220, 1935.

invisible after February 9. The measured K intensities, moreover, do not give high values when the blending would be expected.

The second displaced components of the Fe^+ lines, which emerged about the beginning of January, show at that time a velocity of -760 km., which corresponds to a violet shift of 10 A. at K. Any components of the two lines close to K which have such a displacement will remain clear to the violet side of that line.

There is as yet no definite evidence of any known line which blends with K_I after its emergence from the wing of K_S. No more can be said on the matter until a complete analysis of the spectrum has been made.

In deriving a weighted mean intensity for the interstellar line the fifteen plates taken on and after January 22 have alone been used. These give for the total absorption $10{\cdot}00 \pm 0{\cdot}41$.

4. *Radial Velocity.*—The mean radial velocity of the detached lines has been found by giving half weight to values for H, and by omitting both the first plate where blending occurs with an iron line and the H measure of February 26. The result, from 19 measures, is $-21{\cdot}4 \pm 1{\cdot}6$ km./sec. The component of the solar motion in the direction of the nova is $-19{\cdot}2$ km./sec.

5. *Interstellar H Line.*—The interstellar H line is also visible on most of the plates, but no attempt has been made to measure its intensity unless it was plainly visible and clear of the strong stellar absorption, which is a blend of H and the hydrogen line $H\epsilon$. There is one α Cygni line, λ 3974·17 Fe^+, which is usually seen closely adjacent on the long wave-length side of H. The separation between the two lines decreases as the Fe^+ displacement increases, but the two lines are still seen separately on the plate of March 12, when the intensity of λ 3974 is only about one-quarter that of H. The large width of the only H line measured after this date may be due to this line blending with it, or, more probably, to the emergence of the displaced O^+ line λ 3973·27.

The weighted mean intensity of H from eight plates is $5{\cdot}00 \pm 0{\cdot}32$, or, omitting the last plate, $4{\cdot}63 \pm 0{\cdot}19$. When the wing correction discussed in the next section is applied to the latter figure the final intensity becomes 6·9. This figure, when combined with that derived for K, yields the ratio $K/H = 1{\cdot}70$; but this procedure is open to objection, because H was only measured on those plates where it was plainly to be seen, *i.e.* we have picked out measures above the average for H to combine with the average for K. Actually there is one plate of very good quality, taken on February 17, on which H is almost invisible and quite unmeasurable although K is quite strong (see Plate). This observation is very puzzling as the line is seen on all the other plates on which it would be expected to show. The photometer tracing also suggests no way in which the line might be masked.

For weak lines the central depth, which is more reliably measured than the width, is proportional to the total absorption. From the twelve plates for which the depth of both H and K has been measured, and assigning the assumed minimum detectable depth of 4 to the plate of February 17, we find $K/H = 2{\cdot}00$. By omitting T 361 and four other plates of rather low weight we get $K/H = 1{\cdot}72$.

It seems probable that the ratio is nearer to 2, as it should be when line intensity is proportional to the number of absorbing atoms, than to the theoretical 1·43 for the case of radiation damping.

6. *Distance from K-line Intensity.*—The galactic latitude of Nova Herculis is 25°, and there are, unfortunately, very few early-type stars in its immediate neighbourhood which show interstellar lines. It is therefore necessary to

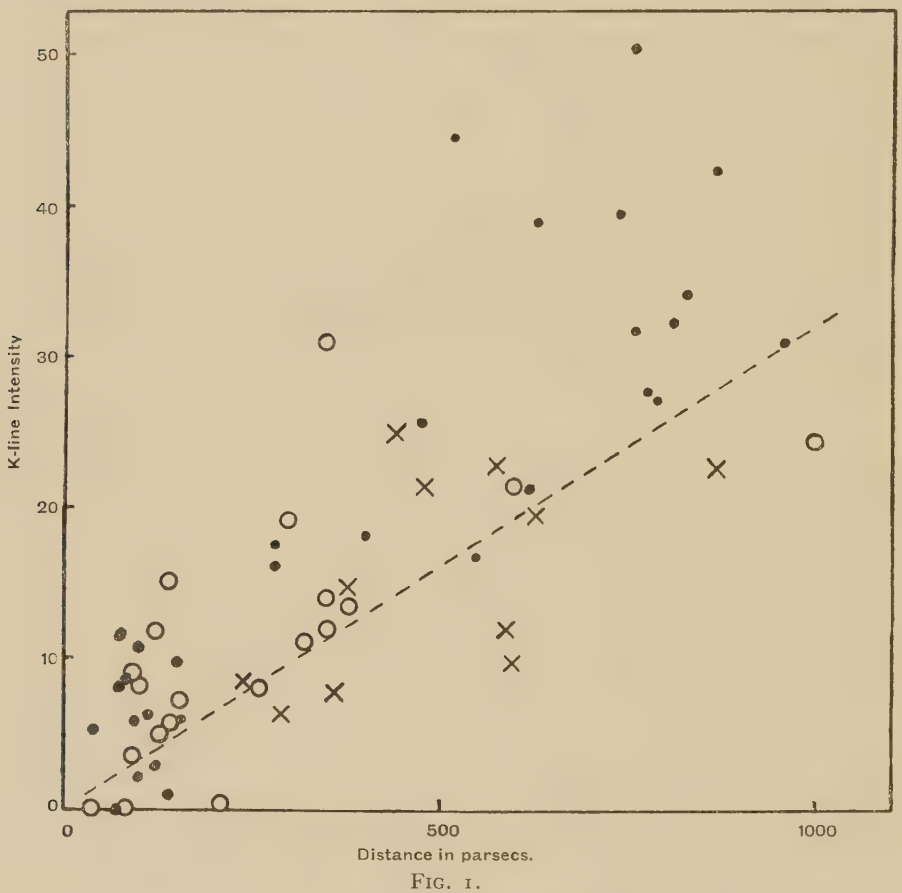


Fig. 1.

employ a distance intensity relation which is based on measures made for stars in all parts of the sky. The most extensive material yet available is the writer's list * of interstellar line intensities for 67 early-type stars. The scale of these Mt. Wilson spectra is only slightly less than that employed in the present work, and emulsions of similar type were used. But before the results are directly comparable it is necessary to apply a positive correction to the observed intensity in the nova to allow for under-estimation of the extent of the wings of the line. In my earlier paper the size of this correction, which depends upon the intensity of the line, was found to be 17 per cent. for a line of strength 10 units. Hence the *K* intensity for Nova Herculis becomes increased from 10·0 to 11·7.

* *Loc. cit.*

In fig. 1 *K*-line intensity is plotted against stellar distance. The distances, which are mostly taken from a paper by Stebbins and Huffer,* are based on the spectroscopic parallax method as applied to B-type stars by Adams and Joy and by Edwards, though the type and line-grade classification of each star are Pearce's. These distances are, of course, subject to considerable uncertainty. Nevertheless when used statistically they should give useful results, and in any case we have no better ones at the present time. All *c* stars in the list, on the suggestion of Merrill,† have had their distances calculated on the assumption that their absolute magnitudes are -5 throughout.

The stars are represented by dots, circles and crosses, according as their distances from the galactic plane deduced from Stebbins and Huffer's distances are less than 30, between 30 and 100, and more than 100 parsecs. It is seen that *K*-line intensity increases least rapidly with distance for the group of stars of highest reduced latitude, to which the nova belongs. The straight line has been drawn to represent the relation for this group. We find $K = 0{\cdot}0317d$ where d is the distance in parsecs. This gives for Nova Herculis $d = 370$ parsecs (1200 light years).

The apparent magnitude of the nova at maximum was 1·3. With the above distance this gives an absolute magnitude of $-6{\cdot}5$, whilst the mean apparent magnitude before outburst, 14·5, corresponds to absolute magnitude $+6{\cdot}7$, which is normal for a K5 dwarf.

The large distance from the galactic plane, about 150 parsecs, is noteworthy.

Apart from the absence of interstellar *H* on plate *T* 361 we may have some confidence in the intensity determined for the interstellar lines, but the derived distance may be in error by as much as 50 per cent., partly because of the uncertainty of spectroscopic parallaxes for B stars, and also on account of possible departures from uniformity in the calcium cloud, especially in high latitudes.

In conclusion I wish to express my thanks to Professor Stratton for his constant helpfulness throughout this investigation.

Summary.

(1) The intensity of the interstellar *K* line has been measured on twenty-four plates. It is found that when the line is blended with the strong nova *K* line the measured intensity is too small. A mean intensity of 11·7 in 0·1 wave-number units is derived from fifteen plates on which the line is clear of stellar absorption.

(2) The material suggests that the ratio of *K* to *H* is abnormally large. *H* has been measured on eight plates, but on one good plate it was almost invisible.

(3) Radial velocity measures made by Professor Stratton confirm the

* *Pub. Washburn Obs.*, **15**, 217, 1934.

† *Ap. J.*, **81**, 354, 1935.

detached character of both *H* and *K*, as the mean value differs by only 2·2 km./sec. from the component of the solar motion.

(4) A distance of 370 parsecs is derived by comparing the strength of interstellar *K* in the nova with its intensity in a number of early-type stars whose distances are given by the spectroscopic parallax method. This distance corresponds to an absolute magnitude of −6·5 at maximum, whilst the mean pre-outburst magnitude is about +6·7.

DESCRIPTION OF THE PLATE

I. Sixteen-fold enlargements of spectra of the *H* and *K* region. The position of the undisplaced lines is indicated by the arrows at the top.

(*a*) Jan. 15. Numerous lines present in the spectrum of *γ* Cygni are seen. *H* and *K* each show two strong absorption components. Interstellar *H* is over-exposed in the reproduction.

(*b*) Feb. 17. On this plate interstellar *H* is extremely weak though *K* is quite strong.

(*c*) Mar. 26. The stellar hydrogen line *Hε* is very complex, whilst the calcium lines are much weaker than on preceding plates. Both interstellar lines are seen.

II. Microphotometer tracings of the *K* line. The complex strong absorption band is due to the stellar calcium line. Interstellar *K* is marked with arrows.

(*d*) Jan. 4. Interstellar *K* is difficult to measure as it lies on the edge of the main *Ca* absorption and also blends with a displaced iron line.

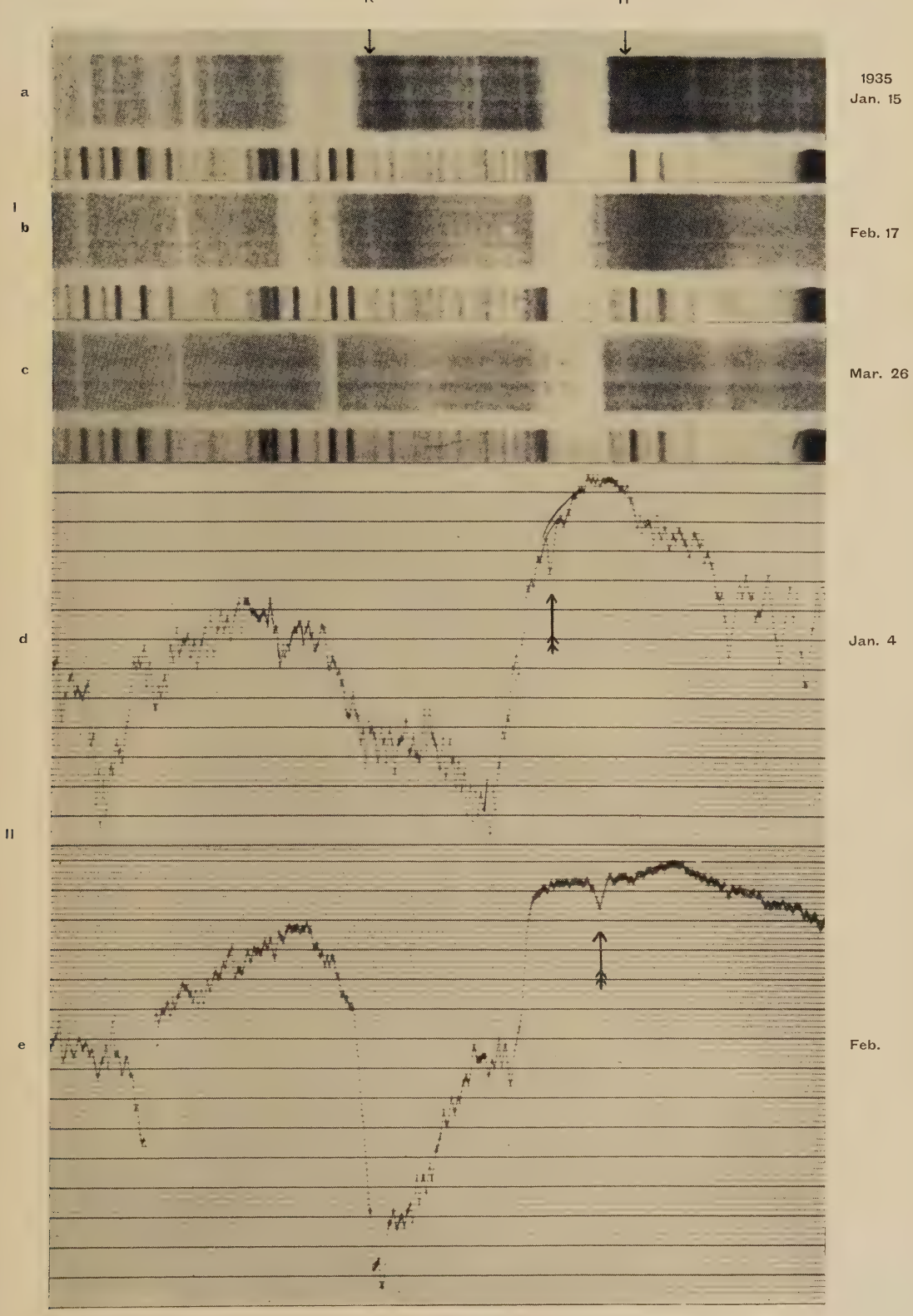
(*e*) Feb. 13. The line is now quite clear of the main absorption and falls on the even background of the broad *K* emission band.

THE BEHAVIOUR OF CARBON, NITROGEN AND SILICON IN THE SPECTRUM OF P CYGNI.

C. S. Beals, D.Sc.

In a previous publication * an account was given of some recent investigations of the spectrum of the star P Cygni. Attention was called particularly to the relation between the ratio of intensities of emission and absorption components of P Cygni lines and the ionization potential of the atom concerned. Reference was also made to the temperature of P Cygni, computed from the lines of different atoms by Zanstra's method, which was found to be in the neighbourhood of 30,000°.

* *Observatory*, **57**, 319, 1934.



E. G. Williams, The Parallax of Nova Herculis 1934.

Recently, observations of the spectrum of P Cygni have been extended to take in certain lines of *N*III, *Si*II and *Si*IV, which were not included in the first publication. Since carbon, nitrogen and silicon are the only atoms appearing in the spectrum of P Cygni which have so far been observed in successive stages of ionization, their behaviour becomes of unusual interest and importance. For this reason it has seemed desirable to publish a separate account of the intensity and velocity relationships shown by the lines of these atoms.

Identification of Lines

For the ions *C*II, *C*III, *N*II, *N*III, *Si*III and *Si*IV the identifications seem clear and unambiguous, the usual strong lines being present with about the expected intensities. Only lines believed to be free from blends

TABLE I

Identification of SiII Lines

λ(Star)	Emission		Absorption		λ(Lab.)	Int.	Series
	Int.	Vel.	Int.	Vel.			
3855·96	19	− 5	...	...	3856·03	8	$sp^2\,{}^2D - 4^2P^\circ$
...	...	...	...	...	3862·59	6	
...	...	...	...	...	4128·05	8	$3^2D - 4^2F^\circ$
...	...	...	...	...	4130·88	10	
5040·59*	17	−33	...	...	5041·13	8	$4^2P^\circ - 4^2D$
5055·31	50	−51	10	−240	5056·17	10	
5956·53*	14	−54	...	...	5957·61	5	$4^2P^\circ - 5^2S$
5978·42	60	−28	10	−423	5978·97	7	
6346·61	87	−23	4	−270	6347·10	10	$4^2S - 4^2P^\circ$
6370·95	43	−19	6	−298	6371·36	8	

* Blended Line.

have been employed, and in the case of *C*II, *C*III and *N*III this has meant the use of a single line.

The identification of *Si*II lines has been rather more uncertain, so that it seems best to review the evidence relating to the identification of lines of this atom in some detail. Table I gives a list of strong lines in the spectrum of *Si*II with both stellar and laboratory wave-lengths and intensities, together with series relationships. The stellar intensities are given in terms of "equivalent widths" in km./sec.

Of the laboratory wave-lengths of *Si*II listed in Table I it would appear that the S – P doublet lines λ 6347·10 and λ 6371·36 would be the most likely to be present in the stellar spectrum, both because lower levels are involved and because the continuous spectrum is presumably less intense in this

region. The next in order should be the D – F doublet λ 4128·05 and λ 4130·88, while other lines would be expected to be considerably less prominent.

There would appear to be little question of the correctness of the identification of the stellar lines λ 6346·61 and λ 6370·95 with *Si*II. The wave-lengths are in accord with prediction within the probable error of measurement and the ratio of doublet intensities (2 : 1) corresponds precisely with the theoretically computed value for an S – P doublet in emission. The absence of the lines λ 4128–30 is puzzling, and throws considerable doubt on the identification of the remaining weaker lines. In this connection it should be pointed out that Merrill * has observed the lines λλ 5056, 5957, 5978, 6347 and 6371 in the spectrum of the star H.D. 207757 (also of P Cygni type), and has been, apparently, convinced of their identity with *Si*II, even though λ 4128 and λ 4130 were absent. It may be that some physical agency not fully understood is operating to alter the relative intensities of the *Si*II lines, but in view of the uncertainty involved it seems best for the purposes of the present discussion to discard all but the well-authenticated pair λ 6346·61 and λ 6370·95.

Intensity and Velocity Relationships

Table II contains data concerning the intensities and velocities of P Cygni lines due to carbon, nitrogen and oxygen in successive stages of ionization. All measurements of intensity have been made relative to the continuous spectrum at the wave-length where the line occurs. All intensities are given in equivalent widths in km./sec. The values of intensity for the emission wave-lengths refer, of course, to the line alone, and do not contain any contribution from the underlying continuous spectrum.

Since all measurements have been made relative to the continuous spectrum, some correction taking into account the energy distribution in the continuous spectrum is necessary in order to arrive at the true values of the emission intensities. Such corrections must involve some definite assumption concerning the energy distribution. Any assumption adopted will necessarily be somewhat arbitrary, since it is well known that the observed colour temperature is in the neighbourhood of 6000°, while the temperature computed by the writer on the basis of Zanstra's method is approximately 30,000°.† Fortunately within the above limits (30,000° and 6000°) the choice of an assumption does not affect the general conclusions of this paper. For the derivation of the corrected values of intensity which are given in Table II, it has been assumed that the energy distribution corresponds to that of a black body at 30,000°, the low value of the observed colour temperature being attributed to causes external to the star (*e.g.* to space reddening due to selective absorption). This assumption may be in error, but in the present state of knowledge it seems as good as any other. Moreover, the use of a high value for the temperature has the advantage that it affords

* *Mt. W. Cont.*, No. 381 ; *Ap. J.*, **69**, 330, 1929.

† *Observatory*, **57**, 319, 1934.

a more severe test of the validity of the intensity relations which are later discussed.

The correcting factor has been taken arbitrarily as unity for λ 5000 and the values for other regions computed accordingly. The directly observed

TABLE II

Behaviour of Carbon, Nitrogen and Silicon

Atom	I.P.	λ Lab.	Emission			Absorption		Series
			Int. Obs.	Int. Corr.	Vel.	Int.	Vel.	
*C*II	24·3	4267·14	7	11	−32	4	−154	$3d^2D - 4f^2F^\circ$
*C*III	47·67	5695·80	1	1	+14	9	− 28	$3^1P^\circ - 3^1D$
*N*II	29·50	3955·85	7	15	−10	8	− 88	$3s^3P^\circ - 3p^1P$
,,	,,	3994·99	19	40	− 8	38	−111	$3s^1P^\circ - 3p^1P$
,,	,,	4447·03	7	10	−14	8	− 95	$3p^1D - 3d^1P^\circ$
,,	,,	4601·49	26	34	−12	20	− 91	$3s^3P^\circ - 3p^3P$
,,	,,	4607·17	14	18	− 8	24	− 92	
,,	,,	4613·88	13	17	−12	14	− 87	
,,	,,	4621·40	15	20	−23	16	− 90	
,,	,,	4630·55	33	43	− 3	39	− 98	
,,	,,	5666·64	14	7	+ 3	38	−104	$3s^3P^\circ - 3p^3D$
,,	,,	5679·56	28	14	+ 1	37	−111	
,,	,,	5686·21	10	5	− 5	14	− 95	
,,	,,	5710·76	9	5	+ 2	15	− 99	
,,	,,	5747·29	2	1	−10	6	− 99	$3s^1P^\circ - 3p^3D$
,,	,,	6482·07	13	5	− 4	19	−126	$3s^1P^\circ - 3p^1D$
*N*III	47·4	4097·31	...	...	...	9	− 47	$3s^2S - 3p^2P^\circ$
*Si*II	16·27	6347·10	87	37	−23	4	−270	$4^2S - 4^2P^\circ$
,,	,,	6371·36	43	18	−19	6	−298	
*Si*III	33·30	4552·61	10	14	+ 6	32	− 86	$4s^3S - 4p^3P^\circ$
,,	,,	4567·83	8	11	− 1	29	− 77	
,,	,,	4574·75	8	11	+ 5	21	− 78	
*Si*IV	44·95	4088·86	...	...	...	19	− 32	$4^2S - 4^2P^\circ$
,,	,,	4116·10	...	...	...	18	− 33	

values of emission intensity have also been included in Table II for purposes of comparison.

Velocities are given for both emission and absorption lines. Series relationships, laboratory wave-lengths and ionization potentials will also be found in the table. The laboratory data have been taken from Miss Moore's * multiplet table, and are originally due to Fowler † (*Si*II, III,

* *Multiplet Table of Astrophysical Interest, Princeton,* 1933.

† *Phil. Trans. Roy. Soc.*, A, **225**, 1, 1925.

IV), Fowler and Selwyn * (*C*II), Edlen † (*C*III), Fowler and Freeman ‡ (*N*II), and Freeman § (*N*III).

A study of Table II indicates for each atom the existence of certain striking relationships, involving both the absorption velocity and the emission/absorption ratio, which appear to be clearly correlated with the state of ionization of the atom concerned. The value of the emission/absorption ratio shows a remarkable decrease with increase of ionization potential. This is particularly marked for silicon, where the ratio decreases from 6·1 for *Si*II to zero for *Si*IV. The decrease in the case of carbon is from 2·8 for *C*II to 0·1 for *C*III, while for nitrogen the ratio decreases from 0·9 for *N*II to zero for *N*III.

Along with changes in the emission/absorption ratio go equally striking changes in the values of the velocities of approach shown by the absorption lines. Again the change is most marked for Silicon, the velocity decreasing from −289 km./sec. for *Si*II to −33 km./sec. for *Si*IV. Carbon and Nitrogen also exhibit marked changes as indicated by the values −154 km./sec. for *C*II to −28 km./sec. for *C*III and −100 km./sec. for *N*II to −47 km./sec. for *N*III. As may be seen from the diagram in the preceding paper,‖ illustrating the formation of P Cygni lines, the absorption velocity corresponds approximately to the actual velocity of ejection. The emission lines also show some evidence of velocity changes, but these changes are of a minor character compared with those shown by the absorption lines.

In a previous publication to which reference has already been made it was suggested that the variation in the emission/absorption ratio could be interpreted as due to a stratification of the nebulous envelope surrounding the star, the atoms of highest ionization potential being found nearest the stellar surface while atoms of low ionization potential occupied shells of larger diameter farther from the star. This stratification was considered to arise as a consequence of the absorption of ultra-violet light in the process of photo-electric ionization. In the innermost shells, owing to the intensity of short wave-length radiation, only atoms of high ionization potential could exist. Further out, the diminution, by absorption, of the intensity of ultra-violet radiation permitted the existence of more easily ionized atoms. The greater relative intensities of emission lines for atoms of low ionization potential was explained as a consequence of the greater diameter of the shells which they occupy.

If this picture of a stratified envelope is a correct one, then the correlation between the absorption velocities and the state of ionization of the absorbing atoms becomes a very significant one. From what has been said about the relative distances of various shells from the stellar surface, combined with the fact that large velocities of approach are associated with low ionization potentials and *vice versa*, there appears reason to believe that we have a

* *Proc. Roy. Soc.*, A, **120**, 312, 1928.
† *Zs. f. Physik*, **72**, 559, 1931.
‡ *Proc. Roy. Soc.*, A, **114**, 662, 1927.
§ *Ibid.*, A, **121**, 318, 1928.
‖ *Observatory*, **57**, 319, 1934.

method of finding out something about the velocity changes which an atom undergoes in its journey outward from the surface of the star. From the fact that this velocity is greater for atoms in the larger shells, it may be deduced that, over the range of ionization which has been observed, the atom moves outward with accelerated velocity.*

This conclusion, and the method by means of which it has been derived, should eventually be of help in determining the nature of the mechanism responsible for ejection. It is hoped in the near future to extend the methods of observation outlined above to other objects of P Cygni type and also to the related group of Wolf-Rayet stars.

In conclusion the writer wishes to thank Miss Kathleen Bancroft for effective help in the measurements and computations of this paper.

Dominion Astrophysical Observatory,
Victoria, B.C. :
1935 *February* 12.

THE EXTERNAL RADII OF STARS OF THE GENERALISED STANDARD MODEL.

N. Fairclough, M.Sc.

1. In "The Analysis of Stellar Structure, II," † Professor Milne defined certain functions associated with Emden's equation and used these in the solution of his equations of fit. Adopting his notation in this paper, β_2 has been set equal to unity and an independent solution made of the equations of fit to give tables of values of external radii, r_1, and $1-\beta_1$, for various chosen values of ω_3. A further set of tables of ω_3 against $1-\beta_1$, for chosen values of M, is obtained by solution of the equation ‡

$$\frac{1}{{\omega_3}^2} = \frac{16(R/\mu)^4}{\pi(\frac{1}{3}a)G^3M^2} \cdot \frac{1-\beta_1}{{\beta_1}^4} \qquad (1)$$

for β_1. The construction of the diagram in fig. 1, from these tables, to exhibit the series of external radii, r_1, against $1-\beta_1$, for constant values of M, is the main purpose of this paper. The results summarised in fig. 1 are exhibited in a different way in fig. 2, where $\log_{10} r_1$ is plotted against $\log_{10} L$ for constant mass M. The bearing of the results on stellar classification will be discussed by Milne on another occasion, and related with Chandrasekhar's latest results (*M.N.*, **95**, 207, 226, 1935).

* A paper by Dr. Otto Struve which has just appeared (*Ap. J.*, **81**, 66, 1935) reaches a practically identical conclusion.

† *M.N.*, **92**, 622, 1932. The present paper is simply the formal continuation of the paper cited, carried out in accordance with the programme outlined in the concluding note.

‡ *M.N.*, **92**, 628, 1932.

2. In two previous papers * the functions associated with Emden's equation have been calculated, and are accordingly not reproduced in Tables Ia to IXa below. Following the method of solution of the equations of fit outlined by Milne in his paper, corresponding values were found for a, $\xi_0\theta(a)$, β_1, and $\xi_0\theta(a)\beta_1^{-\frac{1}{3}}(1-\beta_1)^{-\frac{1}{6}}$, for $\omega_{\frac{3}{2}}=\omega_{\frac{3}{2}}^0=132{\cdot}3885$ and various values of ω_3. In fig. 1 the continuous lines are curves of r_1 against $1-\beta_1$ for values of ω_3. In the radius formula

$$r_1=\frac{(\frac{1}{3}a)^{\frac{1}{2}}}{(\pi G)^{\frac{1}{2}}}\cdot\frac{K}{(R/\mu)^{\frac{2}{3}}}\cdot\frac{\xi_0\theta(a)}{\beta_1^{\frac{1}{3}}(1-\beta_1)^{\frac{1}{6}}} \qquad (2)$$

the following values were used :—

$$\tfrac{1}{3}a=2{\cdot}55\times10^{-15},\qquad G=6{\cdot}66\times10^{-8},\qquad K=3{\cdot}16\times10^{12},$$

$$\frac{R}{\mu}=\frac{1{\cdot}37\times10^{-16}}{3{\cdot}30\times10^{-24}}\quad\text{and}\dagger\quad\mu=2\mu_H,$$

so that

$$r_1=2{\cdot}13\times10^8\cdot\frac{\xi_0\theta(a)}{\beta_1^{\frac{1}{3}}(1-\beta_1)^{\frac{1}{6}}}.$$

Accordingly, in fig. 1 the radius scale has been reduced by a factor $2{\cdot}13\times10^8$.

3. When the appropriate constant values are substituted, equation (1) may be written

$$\frac{1-\beta_1}{\beta_1^4}=0{\cdot}19917\cdot\frac{M^2}{{\omega_3}^2}, \qquad (3)$$

where M is to be written in terms of $\odot$. Values of M were chosen ranging from $\frac{1}{5}\odot$ to $50\odot$, and for each M there were obtained from equation (3) corresponding values of β_1 and ω_3, which are given in Tables Ib to IIIb. Then in fig. 1 points were marked off on the ω_3-curves to correspond with the tabulated values of ω_3 and $1-\beta_1$ for each value of M. By this means curves of r_1 against $1-\beta_1$, for constant mass, were almost completely determined. These curves are shown in fig. 1 by broken lines. Reference to Tables Ia to IXa will show that the pairs of values of β_1 and $\xi_0\theta(a)\beta_1^{-\frac{1}{3}}(1-\beta_1)^{-\frac{1}{6}}$, when ω_3 has values greater than 2·01813, are not sufficient in number to plot with accuracy the ω_3-curves, and consequently the curves of constant mass. This is when the masses are small, that is to say in the approximate region, $0<1-\beta_1<0{\cdot}2$. It became necessary, therefore, to use the A-group of tables to draw curves of a against $1-\beta_1$ for $\omega_3=2{\cdot}119\ldots$, $4{\cdot}036\ldots$, $8{\cdot}073\ldots$, $20{\cdot}182\ldots$, and of a against $\xi_0\theta(a)$ for the same values of ω_3. This interpolation made it possible to construct Tables Ic to Vc, which show values of $1-\beta_1$ against the reduced r_1 for the smaller masses.

4. To complete fig. 1 two important tables, D and E, were required. They give respectively the values of r_1 at $L=0$, and the least radius, r_1, for

* *M.N.*, **92**, 644, 1932; **93**, 40, 1932.

† These calculations were begun some time ago, before the hydrogen contents of the stars had been analysed.

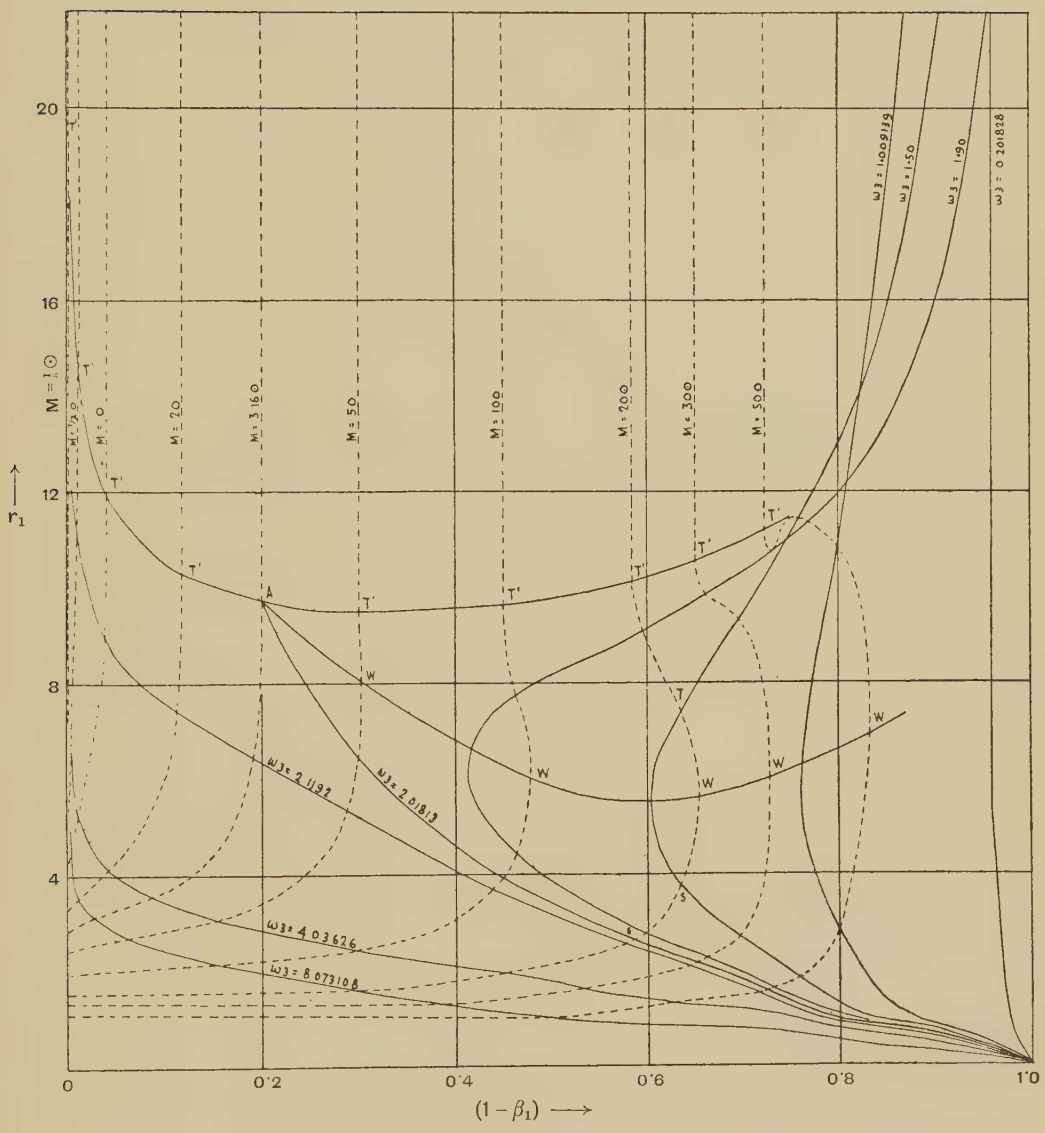


FIG. 1.—*Graphs of r_1 (proportional to external radius) against $(1-\beta_1)$ for constant ω_3 and for constant M. The curve of points T' gives the minimum radius of gaseous configurations; the curve of points W is the locus of maximum luminosity for two-phase configurations.*

$\omega_3 = \omega_3^0$, for each value of M. In "The Analysis of Stellar Structure, I," * Milne arrived at the formula

$$r_1 = \frac{5 \times 5{\cdot}1032}{2^{\frac{2}{5}}\pi^{\frac{2}{5}}} \cdot \frac{K}{GM^{\frac{1}{3}}} \tag{4}$$

for the value of the radius at $1 - \beta_1 = 0$, where

$$K = \tfrac{1}{5} \cdot \left(\frac{3}{4\pi}\right)^{\frac{2}{3}} \cdot \frac{h^2}{(\mu m_H)^{\frac{5}{3}} m_e q_e^{\frac{2}{3}}}.$$

Using the values

$$h = 6{\cdot}55 \times 10^{-27}, \qquad m_H = 1{\cdot}65 \times 10^{-24},$$
$$m_e = 9 \times 10^{-28}, \qquad q_e = 2, \quad \text{and} \quad \mu = 2,$$

we have $K = 3{\cdot}16 \times 10^{12}$, and equation (4) reduces to

$$r_1 = 8{\cdot}8885 \times 10^8 \times M^{-\frac{1}{3}},$$

where again M is to be written in terms of $\odot$. Table D shows the value at $1 - \beta_1 = 0$ of the radius, r_1, for the various masses. In column 3, r_1 is reduced by a factor $2{\cdot}13 \times 10^8$ to conform with the scale of fig. 1. To determine the least radius for $\omega_3 = \omega_3^0$, we used in equation (2), $\theta(a) = 6{\cdot}8969$ and $\xi_0 = 1$, and β_1 had that value given in Tables IB to IIIB for $\omega_3 = 2{\cdot}01813$ for the particular M under consideration. Here again, in Table E, column 4, the numbers are reduced by the factor $2{\cdot}13 \times 10^8$. It will be seen that the critical mass comes out at $3{\cdot}16\odot$. There is apparently a discrepancy between this value and $2{\cdot}62\odot$ given by Milne. This is explained by the fact that throughout these calculations μ has been set equal to 2, whereas he has chosen the value of $2{\cdot}2\ \frac{\mu_H}{\mu}$ as unity.† The value of the critical mass is sensitive to the value of μ, and Table F below has been included to show this variation in the critical mass with variation in μ. The notation in fig. 1 corresponds to the lettering of the diagrams in "The Analysis of Stellar Structure, II." The curve through A joining all points, T′, terminates the purely gaseous parts of the loci ; the curve through A and the W's joins all points of maximum luminosity.

5. In Milne's paper ‡ β_1 is defined by

$$1 - \beta_1 = \frac{\kappa_1 L}{4\pi c G M}.$$

Hence,

$$\log_{10} L = \text{constant} + \log_{10} M + \log_{10} (1 - \beta_1). \tag{5}$$

For the chosen values of M, points on the curves of constant mass in fig. 1 were read off and gave corresponding values of $1 - \beta_1$ and the reduced r_1. By equation (5) we found values of $\log_{10} L$ for values of $1 - \beta_1$, and accordingly constructed tables of $\log_{10} r_1$ and $\log_{10} L$, which are given below, IG to XG. The columns headed $\log_{10} r_1$ are actually values of

* *M.N.*, **91**, 33, 1930.
† *Ibid.*, **92**, 631, 1932.
‡ *Ibid.*, **92**, 616, 1932.

$\log_{10}[r_1 \times (2{\cdot}13 \times 10^8)^{-1}]$, while in columns headed $\log_{10} L$ the value of the constant appearing in equation (5), $\log_{10}(4\pi cG\kappa_1^{-1})$, has not been added. Also values of $\log_{10} M$ are values of $\log_{10} M/\odot$. In fig. 2 these tabulated values of $\log_{10} r_1$ are plotted against the tabulated values of $\log_{10} L$. The mass curves are shown approaching their asymptotic values as $L \to 0$,

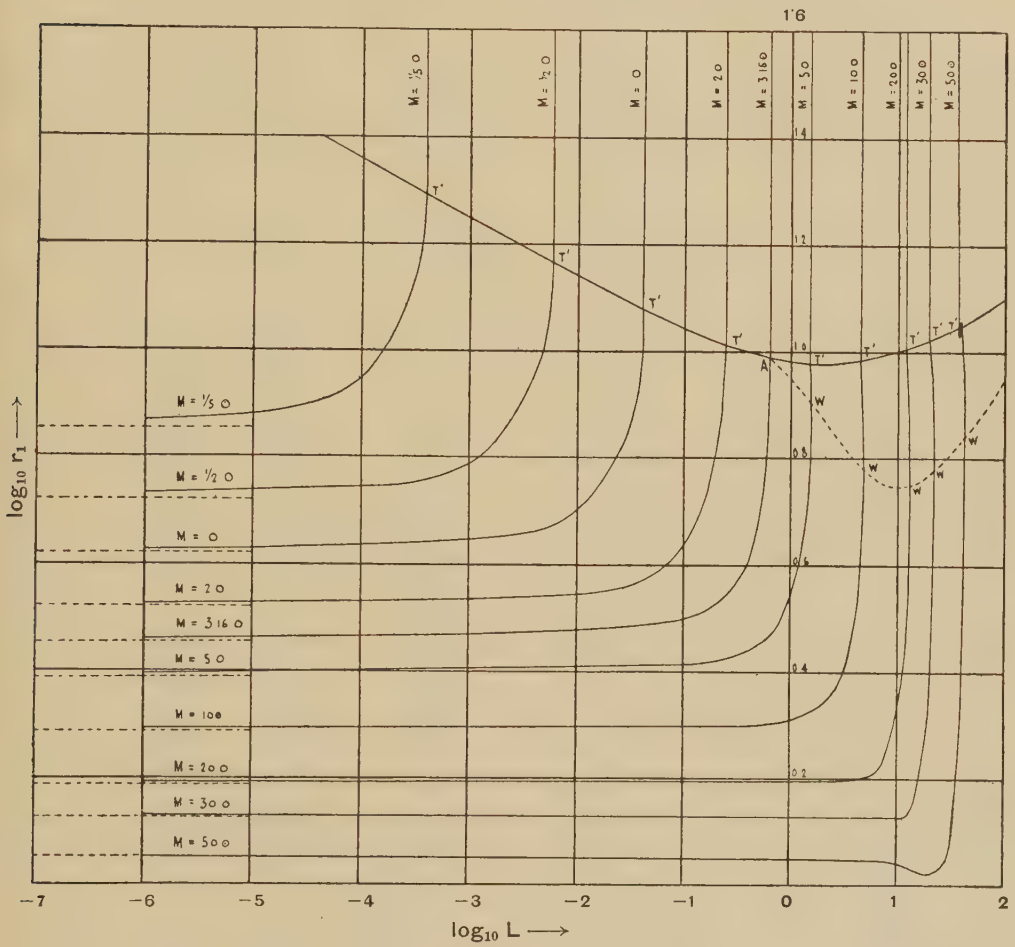


FIG. 2.—*Curves of* $\log_{10} L$ (*constant omitted*) *against* $\log_{10}$ (*external radius*) (*constant omitted*) *for one- and two-phase configurations of constant mass.*

$\log_{10} L \to -\infty$. The curve joining the points, T′, gives the minimum radius of gaseous configurations, and the curve joining the W's is the *locus* of maximum luminosity.

6. These computations were carried out at the request of Professor E. A. Milne, with the aid of a grant made for the purpose by the Department of Scientific and Industrial Research. I wish to express my thanks to Professor Milne for advice and suggestions at various stages in the work, and to Mr. W. Barber, who kindly traced the diagrams from the original pencil copies.

TABLE IA

$(n=\frac{3}{2})$ $\omega_{\frac{3}{2}}^{\frac{3}{2}}=132\cdot3885$ $\eta_0=3\cdot6538$
$(n=3)$ $\omega_3=2\cdot01813$ $\xi_0=1$

a	$\xi_0\theta(a)$	$\beta_1=\frac{8}{5}\cdot\frac{v}{V}$	$\frac{\xi_0\theta(a)}{\beta_1^{\frac{1}{3}}(1-\beta_1)^{\frac{1}{6}}}$
0	6·8969	·800	9·715
·0362	6·8257	·796	9·599
·0725	6·6199	·787	9·278
·1087	6·3005	·780	8·809
·1450	5·8972	·762	8·202
·1740	5·5353	·744	7·666
·2030	5·1550	·727	7·118
·2320	4·7693	·709	6·570
·2610	4·3884	·681	6·034
·2900	4·0197	·656	5·527
·3262	3·5836	·622	4·937
·3625	3·1801	·585	4·403
·3987	2·8115	·549	3·921
·4350	2·4774	·513	3·489
·4712	2·1761	·472	3·109
·5075	1·9052	·434	2·767
·5437	1·6619	·396	2·462
·5800	1·4433	·357	2·190
·6162	1·2466	·322	1·941
·6525	1·0694	·296	1·701
·6887	·9093	·270	1·483
·7250	·7642	·248	1·276
·7612	·6324	·228	1·081
·7975	·5123	·195	·916
·8337	·4024	·135	·804
·8700	·3017	·105	·651
·9062	·2089	·0702	·513
·9425	·1232	·0463	·346
1·0000	0	0	0

TABLE IIA

$(n=\frac{3}{2})$ $\omega_{\frac{3}{2}}^{\frac{3}{2}}=132\cdot3885$ $\eta_0=3\cdot6538$
$(n=3)$ $\omega_3=1\cdot009139$ $\xi_0=1$

a	$\xi_0\theta(a)$	$\beta_1=\frac{8}{5}\cdot\frac{v}{V}$	$\frac{\xi_0\theta(a)}{\beta_1^{\frac{1}{3}}(1-\beta_1)^{\frac{1}{6}}}$
1·00	0	0	0
·95	·0531	...	...
·90	·1121	·0317	·356
·80	·2523	·0640	·638
·70	·4322	·113	·912
·60	·6711	·149	1·300
·50	1·0016	·172	1·859
·40	1·4838	·195	2·653
·35	1·8158	·209	3·182
·30	2·2406	·224	3·849
·25	2·8009	·235	4·746
·20	3·5711	·238	6·030
·15	4·6988	·227	8·041
·10	6·5512	·195	11·713
·08	7·7203	·176	14·228
·06	9·4209	·150	18·218
·04	12·3112	·119	25·563
·02	19·4293	·0841	45·000
·01	31·6539	·0670	78·842
·005	53·9319	·0566	141·84
·003	81·9587	·0526	220·72
·001	219·196	·0473	611·03
·0005	409·901	·0437	1172·4
0	∞	0	∞

TABLE IIIA

$(n=\frac{3}{2})$ $\omega_{\frac{3}{2}}=132{\cdot}3885$ $\eta_0=3{\cdot}6538$
$(n=3)$ $\omega_3=1{\cdot}50$ $\xi_0=1$

a	$\xi_0\theta(a)$	$\beta_1=\frac{8}{5}\cdot\frac{v}{V}$	$\frac{\xi_0\theta(a)}{\beta_1^{\frac{1}{3}}(1-\beta_1)^{\frac{1}{6}}}$
1	0	0	0
·95	·0789	...	...
·90	·1667	·0542	·445
·80	·3749	·109	·800
·70	·6419	·190	1·157
·60	·9945	·232	1·691
·50	1·4754	·281	2·380
·45	1·7841	·315	2·793
·40	2·1537	·345	3·295
·35	2·6000	·372	3·907
·30	3·1431	·391	4·669
·25	3·8091	·397	5·639
·225	4·1988	·393	6·230
·20	4·6338	·378	6·937
·175	5·1219	·356	7·777
·15	5·6746	·322	8·833
·125	6·3113	·276	10·230
·10	7·0700	·220	12·207
·08	7·8212	·171	14·538
·06	8·8180	·125	18·033
·04	10·3997	·0834	24·151
·02	14·2344	·0507	38·794
·01	21·0633	·0392	62·420
·005	34·0545	·0327	107·09
·003	50·9678	·0288	167·09
·001	133·663	·0211	485·43
0	∞	0	∞

TABLE IVA

$(n=\frac{3}{2})$ $\omega_{\frac{3}{2}}=132{\cdot}3885$ $\eta_0=3{\cdot}6538$
$(n=3)$ $\omega_3=1{\cdot}90$ $\xi_0=1$

a	$\xi_0\theta(a)$	$\beta_1=\frac{8}{5}\cdot\frac{v}{V}$	$\frac{\xi_0\theta(a)}{\beta_1^{\frac{1}{3}}(1-\beta_1)^{\frac{1}{6}}}$
1	0	0	0
·95	·1000	·0374	·301
·90	·2111	·0748	·508
·80	·4748	·176	·875
·70	·8124	·246	1·359
·60	1·2556	·309	1·975
·50	1·8503	·404	2·728
·40	2·6580	·495	3·765
·35	3·1630	·538	4·423
·30	3·7432	·571	5·195
·25	4·3974	·589	6·084
·20	5·1129	·574	7·092
·15	5·8627	·477	8·359
·10	6·6175	·255	10·960
·08	6·9255	·158	13·183
·06	7·2625	·0858	16·714
·04	7·7077	·0426	22·230
·02	8·6845	·0196	32·317
·01	10·4286	·0126	44·911
·006	12·6864	·00790	63·783
·002	24·4497	·00499	143·20
·001	41·278	·00419	256·22
·0005	74·886	·00380	480·19
·0003	119·37	·00366	775·06
·0001	353·15	·00358	2309·9
0	∞	0	∞

TABLE Va

$(n=\frac{3}{2})$ $\omega_{\frac{3}{2}}=132\cdot3885$ $\eta_0=3\cdot6538$
$(n=3)$ $\omega_3=2\cdot11920$ $\xi_0=1$

a	$\xi_0\theta(a)$	$\beta_1=\frac{8}{5}\cdot\frac{v}{V}$	$\frac{\xi_0\theta(a)}{\beta_1^{\frac{1}{3}}(1-\beta_1)^{\frac{1}{6}}}$
1	0	0	0
·95	·1115	·0429	·321
·90	·2355	·0858	·542
·85	·3739	·129	·757
·80	·5295	·202	·937
·75	·7055	·248	1·178
·70	·9056	·279	1·464
·65	1·1345	·318	1·772
·60	1·3974	·360	2·116
·55	1·7006	·399	2·515
·50	2·0506	·476	2·925
·45	2·4542	·537	3·433
·40	2·9166	·598	4·030
·35	3·4395	·659	4·729
·30	4·0174	·729	5·549
·25	4·6318	·826	6·607
·20	5·2442	1·069	...

TABLE VIa

$(n=\frac{3}{2})$ $\omega_{\frac{3}{2}}=132\cdot3885$ $\eta_0=3\cdot6538$
$(n=3)$ $\omega_3=4\cdot03626$ $\xi_0=1$

a	$\xi_0\theta(a)$	$\beta_1=\frac{8}{5}\cdot\frac{v}{V}$	$\frac{\xi_0\theta(a)}{\beta_1^{\frac{1}{3}}(1-\beta_1)^{\frac{1}{6}}}$
1	0	0	0
·95	·2124	·101	·464
·90	·4484	·205	·790
·85	·7119	·361	1·077
·80	1·0073	·439	1·459
·75	1·3392	·534	1·875
·70	1·7119	·683	2·354
·65	2·1281	·850	3·082
·60	2·5871	1·064	...

TABLE VIIa

$(n=\frac{3}{2})$ $\omega_{\frac{3}{2}}=132\cdot3885$ $\eta_0=3\cdot6538$
$(n=3)$ $\omega_3=8\cdot07311$ $\xi_0=1$

a	$\xi_0\theta(a)$	$\beta_1=\frac{8}{5}\cdot\frac{v}{V}$	$\frac{\xi_0\theta(a)}{\beta_1^{\frac{1}{3}}(1-\beta_1)^{\frac{1}{6}}}$
1	0	0	0
·95	·4249	·257	·702
·90	·8967	·596	1·239
·85	1·4218	·811	2·013
·80	2·0043	1·182	...

TABLE VIIIa

$(n=\frac{3}{2})$ $\omega_{\frac{3}{2}}=132\cdot3885$ $\eta_0=3\cdot6538$
$(n=3)$ $\omega_3=20\cdot18277$ $\xi_0=1$

a	$\xi_0\theta(a)$	$\beta_1=\frac{8}{5}\cdot\frac{v}{V}$	$\frac{\xi_0\theta(a)}{\beta_1^{\frac{1}{3}}(1-\beta_1)^{\frac{1}{6}}}$
1	0	0	0
·95	1·0621	1·027	...
·90	2·2373	1·970	...

TABLE IXA

$(n=\frac{3}{2})$ $\omega_{\frac{3}{2}}=132{\cdot}3885$ $\eta_0=3{\cdot}6538$
$(n=3)$ $\omega_3=\ \ {\cdot}201828$ $\xi_0=1$

a	$\xi_0\theta(a)$	$\beta_1=\frac{8}{5}\cdot\frac{v}{V}$	$\frac{\xi_0\theta(a)}{\beta_1^{\frac{1}{3}}(1-\beta_1)^{\frac{1}{6}}}$
1	0	0	0
·9	·02243	...	...
·8	·05046	·00727	·261
·7	·08650	·0112	·388
·6	·1345	·0150	·547
·5	·2018	·0186	·763
·4	·3025	·0225	1·076
·3	·4700	·0263	1·587
·2	·8032	·0305	2·583
·1	1·7903	·0386	5·331
·08	2·2780	·0395	6·733
·06	3·0842	·0399	9·084
·04	4·6779	·0402	13·747
·03	6·2531	·0401	18·393
·02	9·3660	·0400	27·573
·01	18·5145	·0391	54·917
·006	30·463	·0384	90·870
·002	88·508	·0369	267·59
·001	173·14	·0358	528·68
·0005	338·80	·0343	1048·9
0	∞	0	∞

TABLE IB

ω_3	$M=\frac{1}{5}\odot$ $1-\beta_1$	$M=\frac{1}{2}\odot$ $1-\beta_1$	$M=\odot$ $1-\beta_1$	$M=2\odot$ $1-\beta_1$
20·18277	·00002	·00012	·00049	·00194
8·073108	·00012	·00076	·00302	·01166
4·03626	·00049	·00302	·01167	·04131
2·11920	·00176	·01062	·03799	·11087
2·01813	·00194	·01167	·04131	·11824
1·90	·00219	·01309	·04575	·12775
1·50	·00349	·02038	·06706	·16892
1·009139	·00759	·04130	·11823	·24894
·201828	·11823	·29758	·44939	·58426

Table IIb

ω_3	$M=3{\cdot}16\odot$ $1-\beta_1$	$M=5\odot$ $1-\beta_1$	$M=10\odot$ $1-\beta_1$
20·18277	·00479	·01166	·04130
8·073108	·02731	·05972	·15546
4·03626	·08541	·15547	·29759
2·11920	·19032	·28682	·43907
2·01813	·2	·29759	·44941
1·90	...	·31094	·46205
1·50	...	·36342	·51006
1·009139	...	·44939	·58426
·201828	...	·72271	·79895

Table IIIb

ω_3	$M=20\odot$ $1-\beta_1$	$M=30\odot$ $1-\beta_1$	$M=50\odot$ $1-\beta_1$
20·18277	·11823	·18971	·29758
8·073108	·29758	·38738	·49538
4·03626	·44941	·53124	·62226
2·11920	·57558	·64365	·71647
2·01813	·58427	·65123	·72272
1·90	·59482	·66040	·73025
1·50	·63417	·69445	·75807
1·009139	·69319	·74494	·79895
·201828	·85539	·88105	·90718

Table Ic

$M=\frac{1}{5}\odot$

ω_3	$1-\beta_1$	a	$\xi_0\theta(a)$	$\dfrac{\xi_0\theta(a)}{\beta_1^{\frac{1}{3}}(1-\beta_1)^{\frac{1}{6}}}$
20·18277	·00002	·952	1·04	6·313
8·073108	·00012	·823	1·73	7·790
4·03626	·00049	·610	2·49	8·870
2·11920	·00176	·205	5·18	14·902

TABLE IIc

$M = \frac{1}{2} \odot$

ω_3	$1 - \beta_1$	a	$\xi_0\theta(a)$	$\frac{\xi_0\theta(a)}{\beta_1^{\frac{1}{3}}(1-\beta_1)^{\frac{1}{6}}}$
20·18277	·00012	·952	1·04	4·683
8·073108	·00076	·823	1·73	5·728
4·03626	·00302	·611	2·49	6·543
2·11920	·01062	·206	5·16	11·045

TABLE IIIc

$M = \odot$

ω_3	$1 - \beta_1$	a	$\xi_0\theta(a)$	$\frac{\xi_0\theta(a)}{\beta_1^{\frac{1}{3}}(1-\beta_1)^{\frac{1}{6}}}$
20·18277	·00049	·952	1·04	3·710
8·073108	·00302	·824	1·73	4·542
4·03626	·01167	·613	2·48	5·217
2·11920	·03799	·210	5·13	8·954

TABLE IVc

$M = 2 \odot$

ω_3	$1 - \beta_1$	a	$\xi_0\theta(a)$	$\frac{\xi_0\theta(a)}{\beta_1^{\frac{1}{3}}(1-\beta_1)^{\frac{1}{6}}}$
20·18277	·00194	·952	1·04	2·947
8·073108	·01166	·825	1·70	3·584
4·03626	·04131	·620	2·39	4·123
2·11920	·11087	·226	4·94	7·412

TABLE Vc

$M = 3{\cdot}16\,\odot$

ω_3	$1-\beta_1$	a	$\xi_0\theta(a)$	$\dfrac{\xi_0\theta(a)}{\beta_1^{\frac{1}{3}}(1-\beta_1)^{\frac{1}{6}}}$
20·18277	·00479	·952	1·04	2·537
8·073108	·02731	·827	1·68	3·090
4·03626	·08541	·633	2·28	3·540
2·11920	·19032	·257	4·55	6·437
2·01813	·2	...	...	9·715

TABLE D

M	$r_1\times10^{-9}$	$r_1\times(2{\cdot}13\times10^8)^{-1}$
$\frac{1}{5}\,\odot$	1·5199	7·137
$\frac{1}{2}\,\odot$	1·1199	5·258
$\odot$	·8888	4·173
$2\,\odot$	·7055	3·313
$3{\cdot}16\,\odot$	·6057	2·848
$5\,\odot$	·5198	2·441
$10\,\odot$	·4126	1·937
$20\,\odot$	·3275	1·548
$30\,\odot$	·2861	1·343
$50\,\odot$	·2413	1·133

TABLE E

M	$1-\beta_1$	$r_1\times10^{-9}$	$r_1\times(2{\cdot}13\times10^8)^{-1}$
$\frac{1}{5}\,\odot$	·00194	4·1619	19·540
$\frac{1}{2}\,\odot$	·01167	3·0962	14·537
$\odot$	·04131	2·5337	11·896
$2\,\odot$	·11824	2·1864	10·266
$3{\cdot}16\,\odot$	·2	2·0693	9·715
$5\,\odot$	·29759	2·0223	9·495
$10\,\odot$	·44941	2·0477	9·614
$20\,\odot$	·58427	2·1525	10·106
$30\,\odot$	·65123	2·2414	10·524
$50\,\odot$	·72272	2·3778	11·164

TABLE F

$$\frac{M}{\odot} = 2{\cdot}62\left(\frac{2{\cdot}2\mu_H}{\mu}\right)^2 \quad \text{and} \quad \mu_H = 1$$

μ	1	1·1	1·2	1·3	1·4	1·5	1·6	1·7	1·8	1·9	2·0	2·1	2·2
$\frac{M}{\odot}$	12·68	10·48	8·80	7·50	6·47	5·64	4·95	4·39	3·91	3·51	3·17	2·88	2·62

TABLE IG

$M = \frac{1}{8} \odot$

$1 - \beta_1$	r_1	$\log_{10}(1 - \beta_1)$	$\log_{10} r_1$	$\log_{10} L$
0	7·137	$-\infty$	·8535	$-\infty$
·00012	7·790	$\bar{4}$·0792	·8915	$\bar{5}$·3802
·00049	8·870	$\bar{4}$·6902	·9479	$\bar{5}$·9912
·00176	14·902	$\bar{3}$·2455	1·1733	$\bar{4}$·5465
·00194	19·540	$\bar{3}$·2878	1·2909	$\bar{4}$·5888

TABLE IIG

$M = \frac{1}{2} \odot$

$1 - \beta_1$	r_1	$\log_{10}(1 - \beta_1)$	$\log_{10} r_1$	$\log_{10} L$
0	5·258	$-\infty$	·7209	$-\infty$
·00076	5·728	$\bar{4}$·8808	·7580	$\bar{4}$·5798
·00302	6·543	$\bar{3}$·4800	·8158	$\bar{3}$·1790
·01062	11·045	$\bar{2}$·0261	1·0431	$\bar{3}$·7251
·01167	14·537	$\bar{2}$·0671	1·1625	$\bar{3}$·7661

TABLE IIIG

$M = \odot$

$1 - \beta_1$	r_1	$\log_{10}(1 - \beta_1)$	$\log_{10} r_1$	$\log_{10} L$
0	4·173	$-\infty$	·6204	$-\infty$
·00302	4·542	$\bar{3}$·4799	·6573	$\bar{3}$·4799
·01167	5·217	$\bar{2}$·0670	·7175	$\bar{2}$·0670
·03799	8·954	$\bar{2}$·5796	·9520	$\bar{2}$·5796
·04131	11·896	$\bar{2}$·6161	1·0754	$\bar{2}$·6161

TABLE IVG

$M = 2\odot$

$1-\beta_1$	r_1	$\log_{10}(1-\beta_1)$	$\log_{10} r_1$	$\log_{10} L$
0	3·313	$-\infty$	·5202	$-\infty$
·01166	3·584	$\bar{2}$·0669	·5544	$\bar{2}$·3679
·04131	4·123	$\bar{2}$·6161	·6152	$\bar{2}$·9171
·1109	7·412	$\bar{1}$·0447	·8699	$\bar{1}$·3457
·1182	10·266	$\bar{1}$·0729	1·0113	$\bar{1}$·3739

TABLE VG

$M = 3{\cdot}16\odot$

$1-\beta_1$	r_1	$\log_{10}(1-\beta_1)$	$\log_{10} r_1$	$\log_{10} L$
0	2·848	$-\infty$	·4545	$-\infty$
·00479	2·86	$\bar{3}$·6803	·4564	$\bar{2}$·1800
·02731	3·12	$\bar{2}$·4364	·4942	$\bar{2}$·9361
·08541	3·54	$\bar{2}$·9316	·5490	$\bar{1}$·4313
·15	4·67	$\bar{1}$·1761	·6693	$\bar{1}$·6758
·1903	6·46	$\bar{1}$·2795	·8102	$\bar{1}$·7792
·2	9·715	$\bar{1}$·3010	·9874	$\bar{1}$·8007

TABLE VIG

$M = 5\odot$

$1-\beta_1$	r_1	$\log_{10}(1-\beta_1)$	$\log_{10} r_1$	$\log_{10} L$
0	2·441	$-\infty$	·3876	$-\infty$
·05	2·64	$\bar{2}$·6990	·4216	$\bar{1}$·3980
·10	2·81	$\bar{1}$·0000	·4487	$\bar{1}$·6990
·15	3·02	$\bar{1}$·1761	·4800	$\bar{1}$·8751
·20	3·50	$\bar{1}$·3010	·5441	0
·25	4·21	$\bar{1}$·3979	·6243	·0969
·2976	6·40	$\bar{1}$·4737	·8062	·1727
·301	8·00	$\bar{1}$·4786	·9031	·1776
·2976	9·495	$\bar{1}$·4737	·9775	·1727

TABLE VIIG

$M = 10\odot$

$1-\beta_1$	r_1	$\log_{10}(1-\beta_1)$	$\log_{10} r_1$	$\log_{10} L$
0	1·937	$-\infty$	·2872	$-\infty$
·05	1·990	$\bar{2}$·6990	·2989	$\bar{1}$·6990
·10	2·04	$\bar{1}$·0000	·3096	0
·15	2·12	$\bar{1}$·1761	·3263	·1761
·20	2·20	$\bar{1}$·3010	·3424	·3010
·25	2·29	$\bar{1}$·3979	·3598	·3979
·30	2·44	$\bar{1}$·4771	·3874	·4771
·35	2·70	$\bar{1}$·5441	·4314	·5441
·40	3·14	$\bar{1}$·6021	·4969	·6021
·4494	3·92	$\bar{1}$·6526	·5933	·6526
·4616	4·45	$\bar{1}$·6643	·6484	·6643
·476	6·01	$\bar{1}$·6776	·7789	·6776
·4616	7·73	$\bar{1}$·6643	·8882	·6643
·4494	9·614	$\bar{1}$·6526	·9829	·6526

TABLE VIIIG

$M = 20\odot$

$1-\beta_1$	r_1	$\log_{10}(1-\beta_1)$	$\log_{10} r_1$	$\log_{10} L$
0	1·548	$-\infty$	·1897	$-\infty$
·1	1·57	$\bar{1}$·0000	·1959	·3010
·2	1·59	$\bar{1}$·3010	·2014	·6020
·3	1·62	$\bar{1}$·4771	·2095	·7781
·35	1·66	$\bar{1}$·5441	·2201	·8451
·4	1·76	$\bar{1}$·6021	·2455	·9031
·45	1·95	$\bar{1}$·6532	·2900	·9542
·5	2·15	$\bar{1}$·6990	·3324	1·0000
·55	2·40	$\bar{1}$·7404	·3802	1·0414
·575	2·54	$\bar{1}$·7597	·4048	1·0607
·6	2·80	$\bar{1}$·7782	·4472	1·0792
·6345	3·73	$\bar{1}$·8024	·5717	1·1034
·6535	5·60	$\bar{1}$·8152	·7482	1·1162
·6345	7·46	$\bar{1}$·8024	·8727	1·1034
·6	8·73	$\bar{1}$·7782	·9410	1·0792
·5843	10·106	$\bar{1}$·7666	1·0046	1·0676

TABLE IXG

$M = 30 \odot$

$1-\beta_1$	r_1	$\log_{10}(1-\beta_1)$	$\log_{10} r_1$	$\log_{10} L$
0	1·343	$-\infty$	·1281	$-\infty$
·2	1·343	$\bar{1}$·3010	·1281	·7781
·3	1·346	$\bar{1}$·4771	·1290	·9542
·4	1·348	$\bar{1}$·6021	·1297	1·0792
·45	1·38	$\bar{1}$·6532	·1399	1·1303
·5	1·47	$\bar{1}$·6990	·1673	1·1761
·55	1·65	$\bar{1}$·7404	·2175	1·2175
·6	1·86	$\bar{1}$·7782	·2695	1·2553
·65	2·09	$\bar{1}$·8129	·3201	1·2900
·7	2·85	$\bar{1}$·8451	·4548	1·3222
·727	6·00	$\bar{1}$·8615	·7782	1·3386
·7	9·10	$\bar{1}$·8451	·9590	1·3222
·675	9·60	$\bar{1}$·8293	·9823	1·3064
·66	9·66	$\bar{1}$·8195	·9850	1·2966
·6512	10·524	$\bar{1}$·8137	1·0222	1·2908

TABLE XG

$M = 50 \odot$

$1-\beta_1$	r_1	$\log_{10}(1-\beta_1)$	$\log_{10} r_1$	$\log_{10} L$
0	1·133	$-\infty$	·0542	$-\infty$
·1	1·115	$\bar{1}$·0000	·0472	·6990
·2	1·10	$\bar{1}$·3010	·0414	1·0000
·4	1·05	$\bar{1}$·6021	·0212	1·3011
·5	1·07	$\bar{1}$·6990	·0294	1·3980
·6	1·12	$\bar{1}$·7782	·0492	1·4772
·65	1·20	$\bar{1}$·8129	·0792	1·5119
·70	1·38	$\bar{1}$·8451	·1399	1·5441
·75	1·73	$\bar{1}$·8751	·2380	1·5741
·80	2·85	$\bar{1}$·9031	·4548	1·6021
·825	4·66	$\bar{1}$·9165	·6684	1·6155
·833	6·90	$\bar{1}$·9206	·8388	1·6196
·825	9·04	$\bar{1}$·9165	·9562	1·6155
·80	10·96	$\bar{1}$·9031	1·0399	1·6021
·775	11·50	$\bar{1}$·8893	1·0607	1·5883
·75	11·32	$\bar{1}$·8751	1·0539	1·5741
·73	10·72	$\bar{1}$·8633	1·0302	1·5623
·7227	11·164	$\bar{1}$·8589	1·0477	1·5579

Newport, Mon.:
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EPHEMERIS OF THE RADIANT-POINT OF THE ETA AQUARID METEOR STREAM.

R. A. McIntosh.

The Eta Aquarid meteor stream is visible annually in the southern hemisphere in the last few days of April and the first fortnight of May, providing generally a good display. The meteors can be seen in some numbers in the northern hemisphere near the maximum on May 4.

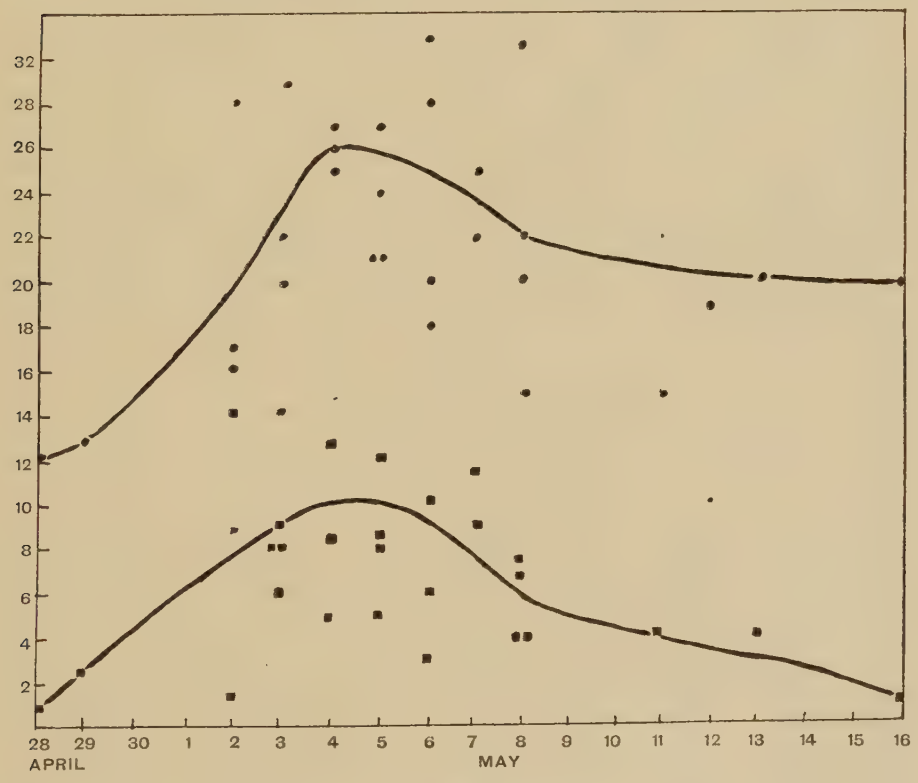

FIG. 1.

During the few years that the Meteor Section of the New Zealand Astronomical Society has been active its members have secured 31 Aquarid radiants, and a stage has been reached where it is possible to employ the results of their observations in forming a preliminary ephemeris of the motion of the radiant-point of the meteor stream.

The horary rate of the Eta Aquarid meteors is shown by the lower curve in fig. 1. This reveals that the meteors, first seen about April 28, show a

gradually increasing rate until maximum is reached about May 4–5, the rate then declining even more slowly until the last meteors of the stream are seen about May 16. The upper curve of the diagram, representing the rate for all meteors (including Aquarids), reveals that a fairly even rate for sporadic meteors is influenced in a marked degree by the presence of the Aquarids.

In constructing the ephemeris of the radiant-point the method described in the author's paper on the Delta Aquarid meteors * has been followed, all the radiants being expressed in ecliptical co-ordinates and their longitudes referred to the meteoric apex. A mean position in these co-ordinates is then derived, which is translated into right ascension and declination for each day of the shower's activity. Under this method the whole of the available data is used in determining the mean radiant, instead of treating each day's observations separately, as has been done in the past.

The tables of radiants which follow are self-explanatory. In the first table are given the usual data, while in the second the radiants corrected for zenith attraction, together with the amounts of the corrections, are shown. These positions are then translated into ecliptical co-ordinates and referred to the meteoric apex, and finally the residuals of each radiant from the mean are shown. An examination of these residuals reveals a very gratifying standard of accuracy in plotting and reducing the radiants. Of the 54 co-ordinates of the 27 radiants used in the discussion only two are greater than 1°·0, while 38 are 0°·5 or less, and of these nine residuals

TABLE I

Radiant-points of Eta Aquarid Meteors

	Date (G.C.T.)	*L*	R.A.	Dec.	Meteors	Wt.	Rate All	Rate Aqds.	Observer
		°	°	°					
1	1931 Apr. 28·68	306·5	330·5	−5·0	2	...	12	1	M
2	1930 ,, 29·69	308·0	330·3	−3·5	5	1	13	2½	M
3	1932 May 2·62	311·1	335·5	−2·5	5	1	16	1½	G
4	1930 ,, 2·69	310·7	334·0	−2·0	19	2	17	9	M
5	1929 ,, 2·70	311·0	334·0	−1·5	18	2	28	14	M
6	1933 ,, 3·60	311·9	336·3	−0·8	8	1	22	8	G
7	1928 ,, 3·69	312·1	336·0	−1·0	5	1	14	6	M
8	1932 ,, 3·68	312·2	335·0	−1·0	16	2	20	8	M
9	1933 ,, 3·69	312·0	335·0	−1·5	12	2	29	9	M
10	1932 ,, 4·65	313·1	337·5	−2·0	34	4	25	8½	G
11	1932 ,, 4·67	313·1	336·5	−1·0	7	1	26	5	M
12	1930 ,, 4·69	312·7	336·5	−1·5	36	4	27	13	M
13	1933 ,, 5·60	313·8	337·5	−0·5	6	1	21	5	G
14	1932 ,, 5·67	314·1	337·5	−0·8	23	3	27	8½	G
15	1929 ,, 5·69	313·9	337·5	−1·0	19	2	21	8	M
16	1930 ,, 5·69	313·5	337·0	−1·0	33	4	24	12	M
17	1929 ,, 6·65	314·8	338·6	0·0	5	1	18	6	M
18	1933 ,, 6·66	314·8	337·5	0·0	5	1	20	3	G

* *M.N.*, **94**, 583, 1934.

	Date (G.C.T.)		*L*	R.A.	Dec.	Meteors	Wt.	Rate All	Rate Aqds.	Observer
			°	°	°					
19	1930 May	6·69	314·8	338·0	−0·5	38	4	28	19	M
20	1932 ,,	6·69	315·2	338·6	−0·6	29	3	31	10	G
21	1930 ,,	7·68	315·8	339·0	0·0	28	3	25	11½	M
22	1929 ,,	7·69	315·8	339·0	−0·4	24	3	22	9	M
23	1932 ,,	8·66	317·0	340·0	0·0	6	1	15	4	M
24	1930 ,,	8·67	316·5	340·0	+0·5	13	2	22	7½	M
25	1932 ,,	8·68	317·0	339·3	+1·0	13	3	31	7	G
26	1929 ,,	8·69	316·8	339·0	+0·3	10	2	20	4	M
27	1931 ,,	11·66	319·2	339·5	+2·0	2	...	...	...	M
28	1929 ,,	11·69	319·7	342·7	+2·5	21	3	15	4	M
29	1932 ,,	12·68	321·0	343·7	+2·5	13	2	19	10	M
30	1931 ,,	13·66	321·2	341·0	+2·0	3	...	20	4	M
31	1932 ,,	16·68	324·9	349·5	+8·5	2	...	20	1	M

TABLE II

	Corrected Radiant R.A.	Dec.	Δ*RA*	ΔDec.	*l*	*l* − *L*	*b*	*O* − *M* Δ*l*	Δ*b*
1	330·9	−4·7	·4	·3	331·3	24·8	+ 6·8	...	...
2	330·7	−3·2	·4	·3	331·6	23·6	+ 8·2	−1·7	0·1
3	336·0	−2·0	·5	·5	337·0	25·9	+ 7·5	−0·6	−0·8
4	334·3	−1·7	·3	·3	335·8	25·1	+ 8·4	−0·2	0·1
5	334·3	−1·2	·3	·3	336·0	25·0	+ 8·8	−0·3	0·5
6	336·9	−0·3	·6	·5	338·5	26·6	+ 8·7	1·3	0·4
7	336·3	−0·7	·3	·3	337·8	25·7	+ 8·5	0·4	0·2
8	335·3	−0·7	·3	·3	336·9	24·7	+ 8·9	−0·6	0·6
9	335·3	−1·2	·3	·3	336·7	24·7	+ 8·5	−0·6	0·2
10	337·9	−1·5	·4	·5	339·0	25·9	+ 7·2	0·6	−0·9
11	336·9	−1·2	·4	·3	338·4	25·3	+ 8·4	0·0	0·1
12	336·9	−1·2	·4	·3	338·2	25·5	+ 7·9	0·2	−0·4
13	337·9	0·0	·4	·5	339·6	25·8	+ 8·6	0·5	0·3
14	337·9	−0·4	·4	·4	339·4	25·3	+ 8·3	0·0	0·0
15	337·8	−0·7	·3	·3	339·2	25·3	+ 8·0	0·0	−0·2
16	337·3	−0·7	·3	·3	338·8	25·3	+ 8·2	0·0	−0·1
17	339·0	0·4	·4	·4	340·8	25·9	+ 8·6	0·6	0·3
18	337·8	0·4	·3	·4	339·6	24·8	+ 9·0	−0·5	0·7
19	338·3	−0·2	·3	·3	339·9	25·1	+ 8·3	−0·2	0·0
20	339·0	−0·3	·4	·3	340·5	25·3	+ 7·9	0·0	−0·4
21	339·4	0·3	·4	·3	341·1	25·3	+ 8·3	0·0	0·0
22	339·3	−0·1	·3	·3	340·9	25·1	+ 8·0	−0·2	−0·2
23	340·4	0·4	·4	·4	342·0	25·0	+ 8·0	−0·3	−0·2
24	340·4	0·9	·4	·4	342·2	25·7	+ 8·5	0·4	0·2
25	339·6	1·4	·3	·4	341·7	24·7	+ 9·3	−0·6	1·0
26	339·3	0·6	·3	·3	341·1	24·3	+ 8·6	−1·0	0·3
27	339·9	2·4	·4	·4	342·4	23·2	+10·1	...	...
28	343·1	2·8	·4	·3	345·5	25·8	+ 9·2	0·5	0·9
29	344·1	2·9	·4	·4	346·5	25·5	+ 8·9	0·2	0·6
30	341·4	2·4	·4	·4	343·8	22·6	+ 9·5	...	...
31	349·9	8·9	·4	·4	...	...	...	...	...

are zero. The reason for the greater accuracy in these observations, compared with the Delta Aquarids described in my previous paper, is found in the fact that the low radiants provided meteors which were generally bright, long-pathed and with slow apparent velocities, the meteors leaving a high percentage of bright, enduring trains.

Only two members of the New Zealand Meteor Section have contributed to the observations under discussion, Mr. M. Geddes, Otekura, and the author.

The mean position of the Aquarid radiant-point was determined after weighting the various observed radiants according to the number of meteors comprising each. Four radiants were rejected as being based on less than four meteors. The mean position derived was :

> Longitudes of radiants minus longitude of meteoric apex, 25°·3.
> Latitude, +8°·3.

The fact that for twelve days (possibly eighteen) the radiants observed are clustered within a degree of a point which, while maintaining constant elongation from the meteoric apex, moves over 11°·5 of right ascension (possibly 16°) clearly establishes the fact, already shown by Professor A. S. Herschel * and Dr. C. P. Olivier,† that the radiant-point of the stream is in motion, the daily movement being 0°·96 in right ascension and 0°·37 in declination.

Using the mean position of the radiant found above, the equatorial co-ordinates for selected dates were calculated and those for intervening dates interpolated to form the ephemeris given below :

Ephemeris of Radiant-point for 1935

1935	R.A.	Dec.	1935	R.A.	Dec.
	°	°		°	°
Apr. 28	330·4	−3·1	May 8	339·3	+0·2
,, 29	331·3	−2·8	,, 9	340·2	+0·6
,, 30	332·2	−2·5	,, 10	341·0	+1·0
May 1	333·1	−2·2	,, 11	341·9	+1·3
,, 2	334·0	−1·9	,, 12	342·8	+1·7
,, 3	334·8	−1·6	,, 13	343·7	+2·1
,, 4	335·7	−1·3	,, 14	344·6	+2·5
,, 5	336·6	−0·9	,, 15	345·5	+2·9
,, 6	337·5	−0·5	,, 16	346·4	+3·3
,, 7	338·4	−0·1			

From the adopted mean position of the radiant at maximum the following elliptical orbit was calculated for the meteor stream, the elements of Halley's comet and of an orbit computed by Hoffmeister ‡ being given for purposes of comparison.

* *M.N.*, **38**, 379, 1878.
† "175 Parabolic Orbits . . ." (*Trans. Am. Phil. Soc.*, N.S., **32**, 10, 1911).
‡ *Astr. Nach.*, **196**, 309, 1913.

	Meteors	Comet 1910II	Hoffmeister
	°	°	°
ω	97·3	111·7	99·7
π	140·9	169·0	144·9
Ω	43·6	57·3	45·2
i	162·2	162·2	162·6
a	17·946	17·946	17·946
e	0·9687	0·9673	0·967
q	0·5616	0·5871	0·595
	1935·0	1910·0	1913(?)

The connection between Halley's Comet and the Eta Aquarid meteor stream is clearly shown by a comparison of the three orbits given above.

All calculations in this paper have been performed with four-figure logarithms, the results being rounded off to tenths of a degree.

Private Observatory,
Epsom, Auckland, N.Z.:
1935 *March* 19.

OCCULTATIONS OF STARS BY THE MOON, OBSERVED AT THE NIZAMIAH OBSERVATORY, HYDERABAD, DURING THE YEAR 1934.

(Communicated by the Director)

No.	Star	Mag.	Phase	Date	G.C.T.		
				1934	h	m	s
1	35 B Arietis	6·4	D	Jan. 22	17	38	15·6
2	A.G. Berlin 585	8·7	D	22	17	47	52·1
3	11 Tauri	6·1	D	24	14	33	22·0
4	A.G. Berlin 685	8·8	D	Feb. 19	14	5	49·5
5	40 Geminorum	6·3	D	24	15	15	15·6
6	40 Cancri	6·5	D	26	14	35	27·3
7	39 Cancri	6·5	D	26	14	39	6·8
8	102 B Cancri	6·5	D	26	14	44	55·3
9	A.G. Berlin 3454	7·2	D	26	14	55	26·1
10	A.G. Berlin 3457	8·0	D	26	14	56	54·0
11	A.G. Berlin 3459	8·0	D	26	15	6	17·1
12	ϵ Cancri	6·3	D	26	15	7	7·4
13	A.G. Berlin 2838	8·4	D	Apr. 20	15	3	25·8
14	A.G. Cambridge 3831	8·6	D	20	15	14	16·4
15	A.G. Berlin 2842	6·8	D	20	15	29	58·5
16	52 Geminorum	6·1	D	20	15	39	38·2
17	A.G. Berlin 3225	8·1	D	21	14	11	14·8
18	A.G. Berlin 3234	7·1	D	21	14	28	38·0
19	X Sagittarii	4·4	D	May 2	19	22	5·9
20	A.G. Berlin 3109	8·7	D	18	14	20	8·9
21	A.G. Berlin 3785	8·7	D	20	14	58	25·6
22	*b* Scorpii	4·7	D	Aug. 18	14	45	46·6
23	49 Sagittarii	5·5	D	Oct. 15	14	36	2·2
24	189 B Sagittarii	6·1	D	Nov. 11	13	24	57·3

No.	Star	Mag.	Phase	Date 1934	G.C.T. h m s
25	λ Piscium	4·6	D	Nov. 16	13 10 52·8
26	λ Capricorni	5·5	D	Dec. 11	14 33 11·2
27	35 B Arietis	6·4	D	16	15 7 54·5
28	ϵ Geminorum	3·2	D	21	16 2 27·3
29	ϵ Geminorum	3·2	R	21	17 32 15·9

All the observations were made by M. K. Bappu with the exception of Nos. 6 to 12 which were observed by T. P. Bhaskaran. The instrument used was the 15-inch Grubb refractor and the times were in all cases noted with a stop-watch; the clock corrections were determined from observations with the transit instrument.

The reductions have been carried out in duplicate by M. V. V. Sastri and S. Aravamudan according to the method adopted by the *Nautical Almanac* office. A correction of $0^h.0025$ has been added to the G.C.T. to allow for the error of 5″ in the mean longitude of the Moon.

No.	χ	ρ	σ′ − σ	Cos (χ − ρ)
	° ′	° ′	″	
1	39 8	68 35	−2·8	+0·871
2	8 36	68 37	−2·0	+0·500
3	10 14	77 16	−1·1	+0·390
4	119 15	70 41	−1·6	+0·662
5	139 42	99 36	−1·0	+0·765
6	56 7	109 25	+1·0	+0·598
7	42 49	109 26	+0·2	+0·397
8	124 36	109 27	−5·2	+0·965
9	172 8	109 29	+1·6	+0·459
10	167 16	109 29	+0·2	+0·533
11	158 35	109 30	−1·1	+0·655
12	150 44	109 31	−1·1	+0·752
13	158 9	101 24	+2·0	+0·548
14	51 27	101 27	+0·9	+0·643
15	114 1	101 30	−2·8	+0·976
16	65 40	101 32	−0·9	+0·810
17	172 52	106 16	−2·0	+0·397
18	89 21	106 20	+2·6	+0·956
19	89 42	88 46	−0·3	+1·000
20	151 56	104 49	−1·6	+0·680
21	60 25	112 49	+4·2	+0·610
22	62 51	100 45	−1·4	+0·789
23	44 15	77 6	−6·2	+0·840
24	80 30	78 45	−0·5	+1·000
25	94 28	63 28	−0·5	+0·857
26	55 10	65 53	−1·5	+0·982
27	31 24	69 56	−2·9	+0·782
28	100 19	98 21	−2·0	+0·999
29	265 5	98 42	+2·9	−0·972

In the case of star No. 14, a proper motion ($\mu_\alpha = -0^s.0319$, $\mu_\delta = -0''.124$, taken from Cincinnati Publications, No. 19) has been applied in bringing the star's position to date.

Nizamiah Observatory,
Hyderabad, India.

OCCULTATIONS OF STARS BY THE MOON, OBSERVED AT THE NATIONAL OBSERVATORY OF ATHENS, 1935 APRIL 6–APRIL 17.

(Communicated by S. Plakidis, D.Sc.)

No.	Star	Mag.	Date	Local Sidereal Time	Obs.	Power
			1935	h m s		
1	B.D. +23° 538	7·5	Apr. 6	9 27 20·8	K	155
2	+23° 537	7·5	,, 6	9 29 2·6	,,	,,
3	+23° 535	7·9	,, 6	9 34 8·2	,,	,,
4	+23° 531	8·5	,, 6	9 34 44·3	,,	,,
5	+23° 536	8·0	,, 6	9 35 4·7	,,	,,
6	+23° 541	3·2	,, 6	9 37 8·7	,,	,,
7	+23° 534	8·0	,, 6	9 37 17·6	,,	,,
8	+25° 922	9·2	,, 8	8 38 49·8	,,	,,
9	+25° 926	9·1	,, 8	8 41 58·3	,,	,,
10	+25° 925	9·5	,, 8	8 43 40·9	,,	,,
11	+25° 918	8·4	,, 8	8 51 2·7	,,	,,
12	+25° 935	9·2	,, 8	9 16 59·4	,,	,,
13	+25° 941	6·8	,, 8	9 48 50·6	A	75
14	+25° 970	8·7	,, 8	10 50 10·7	,,	155
15	+25° 966	8·8	,, 8	11 3 44·1	,,	,,
16	+25° 978	6·6	,, 8	11 22 0·3	,,	75
17	+24° 1343	6·8	,, 9	9 44 6·6	K	155
18	+22° 1716	9·5	,, 10	10 8 57·7	A	,,
19	+22° 1712	9·5	,, 10	10 13 24·0	,,	,,
20	Ph. Paris $7^h\ 28^m$ +22° 456	10·1	,, 10	10 59 29·5	,,	,,
21	B.D. +21° 1638	8·8	,, 10	11 9 1·7	,,	,,
22	+18° 1930	6·5	,, 11	10 42 44·9	,,	75
23	+18° 1931	7·7	,, 11	11 10 8·3	,,	155
24	+18° 1939	8·1	,, 11	12 42 22·9	,,	,,
25	− 5° 3416	6·7	,, 16	10 2 28·2	,,	,,
26	− 5° 3419	8·0	,, 16	10 18 19·2	,,	,,
27	−10° 3570	6·0	,, 17	12 59 0·5	,,	75

Instrument used :

Doridis refractor (Gautier, 400 mm.), *M.N.*, **92**, No. 7.

Phase observed :

Only disappearances at the dark limb.

Remarks :

The times of Nos. 12 and 14 are somewhat uncertain. No. 4 through clouds.

Observers :

G. Adamopoulos (A) and D. Kotsakis (K).

1935 *April* 23.